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Model Development for Damped and Forced Type of Oscillations in Time Series

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ABSTRACT

The periodic motion defines as the motion which repeats after a regular interval of time. The periodic motion in which there is the existence of a restoring force and the body moves along the same path to and fro about a definite point called equilibrium position/mean position is called oscillatory motion. The oscillatory motion could be either linear oscillation or circular oscillation. For example, the oscillation of strings of musical instruments is linear oscillation whilst the oscillation of the simple pendulum of a clock is circular oscillation. A wave is a correlated collection of oscillations. For example, in a wave traveling along a string, each point in the string oscillates back and forth in the transverse direction (perpendicular to the direction of the string); in sound waves, each air molecule oscillates back and forth in the longitudinal direction (the direction in which the sound is traveling). Therefore understanding oscillatory motions is the basis of understanding waves. Oscillatory motions and wave-like patterns are common in time series data as well. For example, the number of infected cases of a disease in epidemiology; species migration in ecology, human blood sugar or blood pressure levels in biology; the harvest of crops in agriculture; behavior of consumer price index in economics; share returns in finance; the number of arrivals to a cultural landscape in tourism management, etc. follow regular or irregular wave-like patterns. The Auto-Regressive Integrated Moving Average (ARIMA), Seasonal Auto-Regressive Integrated Moving Average (SARIMA), Circular Model (CM), and Sama Circular Model (SCM) were successful in modeling such series. The literature revealed that the daily infected cases of Covid 19 show irregular wave-like patterns with; increasing amplitudes, decreasing amplitudes, or both, but none of the existing time series forecasting techniques are capable of capturing them. The pattern of these series is somewhat similar to the pattern of Damped oscillation and Forced oscillation described in Physics.

Hence the authors of the study intended to develop suitable forecasting techniques to model such time series and developed two new stochastic models named; Damped Circular Model (DCM) and Forced Circular Model (FCM). The development of the models was based on the Circular model (which was based on Simple harmonic motion), the theory of Damped and Forced Oscillations, and the Second-order Differential Equations. It is recommended to test the DCM and FCM on real-life data in the fields of epidemiology and others.

Keywords: Circular Model (CM), Damped Oscillation, Forced Oscillation

1. INTRODUCTION

1.1 Background of the Study

Repetitive motions are frequently seen in our daily life. In our childhood, we have enjoyed rocking in a cradle or swinging on a swing. Both these motions are repetitive, the object (cradle or swing) moves to and fro about a mean position. Rise and fall of a marker buoy or a small boat on the sea, up and down motion of a yo-yo, back and forth motion of the piston in a steam engine, to and fro motion of a pendulum clock, movement of strings of a playing guitar are also examples for repetitive to and fro motions. Such periodic motions are known as Oscillations. The movement of your bicycle wheels, the orbital motion of planets in the solar system, etc. are also periodic motions but they are not oscillatory indeed.

Oscillatory Motions and Waves

Study of oscillatory motion is basic to physics; its concepts are required for the understanding of many physical phenomena. The motion in which repeats after a regular interval of time is defined as periodic motion. The periodic motion in which there is the existence of a restoring force and the body moves along the same path to and fro about a definite point called equilibrium position/mean position is called oscillatory motion (Vedantu, n.d). The oscillatory motion could be either linear oscillation or circular oscillation. For example, the oscillation of strings of musical instruments is linear whilst the oscillation of the simple pendulum of a clock is circular.

A wave is a correlated collection of oscillations (Morin, n.d). For example, in a wave traveling along a string, each point in the string oscillates back and forth in the transverse direction (perpendicular to the direction of the string); in sound waves, each air molecule oscillates back and forth in the longitudinal direction (the direction in which the sound is traveling). Therefore understanding oscillatory motions is the base of understanding waves. There are three main types of oscillations; Free oscillation, Damped oscillation, and Forced oscillation. They are described by the parameters; period of oscillation, frequency of oscillation, and amplitude of oscillation.

The time taken by an oscillating body to complete one cycle of motion is termed its period. It is generally measured in seconds and is denoted by T. The number of oscillations a body can complete in one second is known as its frequency of oscillation. It is primarily expressed in the SI unit of Hertz and is denoted by the letter *f*. The maximum amount of displacement of an oscillating body from its central position is known as its amplitude. Its value is measured in meter and it is denoted by *A*. Hence oscillatory motions are expressed by the trigonometric functions; sine and cosine.

1.1.1 Free Oscillation

A free oscillation is a motion with a natural frequency of the particle and constant amplitude, energy, and period (VSSUT, n.d).A particle's motion is not under the influence of any external resistance(VSSUT, n.d). An example of free oscillations is the motion of a simple pendulum in a vacuum (VSSUT, n.d).

1.1.2 Damped Oscillation

A damping oscillation is one in which the moving particle gradually loses its kinetic energy on interaction with resistive forces like air or friction(VSSUT, n.d).Due to this resistance offered by external forces, the displacement of a particle slowly reduces with time and ultimately reaches its state of rest(VSSUT, n.d). A simple oscillating pendulum under natural conditions is an example of damped oscillations (VSSUT, n.d).

1.1.3 Forced Oscillation

Forced oscillations in a particle are a result of a continuous application of an external force to help it maintain a motion of constant amplitude, time, and frequency (VSSUT, n.d). A movement embodying forced oscillations definition is vibrations in a loudspeaker induced with the current (VSSUT, n.d).When swinging on a swing a child uses his feet to move the swing is an example for the forced oscillation.

1.2 Research Problem

Mathematical models are developed to explain the wave patterns come across in real life. Mostly these applications are in Physics, Engineering, Geophysics, Oceanography, Atmospheric science, Astronomy etc. For examples, motion of a pendulum, swing, strings of musical instruments, springs of shock absorbers, sea waves, electrical signals, image processing etc. are modelled with the help of Newton's laws of motion, Calculus, De-Mover's theorem and Spectral analysis/ Fourier transformation.

Wave like patterns are common in time series data as well. For examples, number of infected cases of a disease in epidemiology; species migration in ecology, human blood sugar or blood pressure levels in biology; harvest of crops in agriculture; behavior of consumer price index in economics; share returns in finance; number of arrivals to a cultural landscape in tourism management etc. follow regular or irregular wave like patterns. Yet the development and applications of mathematical models in those fields are extremely limited.

Today the entire world is facing the challenge of global infectious disease, Covid-19. Academicians and researchers attempt to forecast the number of daily infected cases, recoveries and deaths to mitigate the risk.

As per Cazelles et al. (2007), Classical Time-Series approaches are widely applied in modelling infected diseases, but not highly successful due to the complex and non-stationary nature of the epidemiological data series. Konarasinghe, (2020) has shown the

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applicability of time series models; exponential smoothing and ARIMA in forecasting Covid-19 outbreak, yet Konarasinghe, (2021) revealed that the situation is different by now. Accordingly the daily infected cases of many countries follow non-stationary wave like patterns, hence the classical time series approaches are not suitable for forecasting them. Also Konarasinghe, (2021) has concluded that the Sama Circular Model (SCM) is highly successful in capturing the wave like pattern of daily infected cases. The SCM is a statistical model based on the frequency domain (Konarasinghe, W. G. S., 2018). It is developed to model the free oscillatory type of patterns in time series; capable in capturing trend, seasonal, cyclical components and auto correlation of any series. Yet the SCM may not be highly successful in damped or forced type of a waves. On the other hand, none of the existing time series forecasting techniques are capable in capturing damped or forced type of a waves

The preliminary data analysis of the present study revealed that the daily infected cases of Covid-19 in many countries follow damped and/ or forced type of oscillations. This may be true for the wave like patterns seen in other fields of studies as well. Hence it is essential to do develop mathematical models to capture them.

Mathematical models can be classified in many ways. Some of them are; Static vs. Dynamic models and Deterministic vs. Stochastic models. A model is said to be "Static" when it does not have time- dependent component. In contrast, dynamic models contain time-dependent component. Deterministic models are not associated with any randomness whilst the stochastic models does. The stochastic models are more powerful than the other type of mathematical models in real life applications. Stochastic models also known as "Statistical Models". Hence the study is aimed on developing Stochastic models.

1.3 Objective of the Study

i. To develop stochastic model to forecast damped oscillatory type of motion in real life

ii. To develop stochastic model to forecast forced oscillatory type of motion in real life

1.4 Significance of the Study

A wave like pattern is common in real life time series data sets. This pattern may contain trend, seasonal, cyclical and irregular variations (Konarasinghe, W.G.S., 2020-a). A trend is the overall tendency of the data series. It can be either upward or downward following linear or non-linear patterns. Seasonal variation is a wave like pattern with period of oscillation less than 12 months whilst a cyclical variation is a wave like pattern with period of oscillation greater or equals to 12 months. When the trend is combined with the seasonal and cyclical components, a time series may follow a wave of a free oscillatory motion, damped oscillatory motion or forced oscillatory motion. In other words, a series may swing with a constant amplitude, decreasing amplitude, increasing amplitude or combinations of them. The Circular Model (CM) of Konarasinghe W.G.S (2016) can be used to model a wave with a constant amplitude whilst the SCM of Konarasinghe, W.G.S. (2018) is capable in capturing a wave with variable amplitudes. As per the literature, CM and SCM have outperformed other time series techniques in many real life

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applications (Konarasinghe, 2021; Konarasinghe, W.G.S., 2020-b; Konarasinghe, W.G.S., 2019), but CM and SCM also helpless if the wave fade off or keep step.

Accurate and reliable forecasting is a lighthouse for any field. It helps for planning, strategy development, and effective and efficient management. Konarasinghe, (2021) says that "forecasting daily infected cases of Covid-19 gives perfect guidelines to control the outbreak of the pandemic. It can be used to control the movements of the general public, preparing of an effective lockdown schedule, preparing of transportation schedules for the essential services". Cazelles et al. (2007) also emphasized the importance of forecasting in controlling infected diseases. Helena et al. (2010) shows the importance of forecasting economic variables whilst explaining the consequences to the economy of not having accurate forecasts. If the complex nature of time series prevents reliable and accurate forecasting, then society becomes helpless. Hence this study would fill the knowledge gap and help society in various ways.

2. LITERATURE REVIEW

Mathematical modeling is the key to forecasting wave-like patterns in real life. A mathematical model is a simplification of a real-world situation into an equation or a set of equations. A model is a description of a system using mathematical concepts and language. According to literature, wave-like patterns were modeled by deterministic models as well as stochastic models, hence the literature review is divided into two parts;

2.1 Studies based on deterministic models

2.2 Studies based on stochastic models

2.1 Studies based on deterministic models

The study of Sujito et al. (2021) is focused on describing the oscillation of a block of mass attached to a light horizontal spring, fixed to a non-movable wall. Authors have first considered the oscillation of the mass on a smooth surface and then on a rough surface. Both oscillations were modeled by second-order differential equations; first the simple harmonic motion and the latter the damped oscillation. They have used Simulation programs using MATLAB software to solve the differential equations and successfully estimated the variables; amplitude, angular frequency, horizontal shift, and vertical shift of the motions.

In topography, a drainage divide is elevated terrain that separates neighboring drainage basins. Stark (2010) applied numerical integration to understand the oscillatory motions of drainage divides. The researcher has derived four ordinary differential equations for variables; height $h(t)$ and displacement $x_0(t)$ of the divide and $x(t)$ and $y(t)$ for the 2-D motion of mass-wasting and solved them by explicit Runge-Kutta method.

Grossman (1980) has studied the oscillatory phenomena in a model of infectious diseases. Accordingly, oscillations of the number of cases around an average endemic level are common in infectious diseases. The author has derived several deterministic models for variables; periodically varying contact rates or from the combined effect of large initial perturbation, small periodic variation of the contact rate, and the destabilizing

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nature of infectious and latent periods when described as time delays and found the solutions for them by Numerical methods.

2.2 Studies based on stochastic models

Sen & Chaudhuri (2016) and Emenike (2010) used the Decomposition techniques in forecasting stock market indices of the Auto sector of the Indian stock market. The authors have estimated the Trend, Seasonal and Irregular components of the series for the data from January 2010 to December 2015 and concluded that the decomposition technique is suitable for the purpose. Prem & Rao (2015) have tested Decomposition models for forecasting wind speed. They have compared the forecasting ability of the Decomposition models with Exponential Smoothing Techniques, ARIMA, and some others and concluded that the Decomposition models are superior to others. However, Sen & Chaudhuri (2016) and Prem & Rao (2015) do not attempt to capture the cyclical variation of the series. Konarasinghe (2016) used Decomposition techniques to forecast tourist arrivals to Sri Lanka from the Western European Region. The author has done the model testing by using monthly data for the period from January 2008 to December 2014 and concluded that the Additive Decomposition models are the best for the purpose.

The study of Prapanna et al. (2014) and Emenike (2010) tested ARIMA on forecasting share returns of the Indian stock market and Nigerian Stock Exchange (NSE) respectively, and found that the ARIMA is successful for the purpose. Konarasinghe, W.G.S. (2016) developed a new forecasting technique, namely "Circular Model (CM)" to capture the wave-like patterns of share returns of the Sri Lankan share market. Konarasinghe et al. (2016) have tested the ARIMA & CM on Forecasting Sri Lankan Share Market Returns and concluded that both methods are suitable for the purpose, but the CM is superior to ARIMA in light of that.

3. METHODOLOGY

The model development is based on the Circular Model (Konarasinghe, W.G.S., 2016), the theory of oscillations in Physics, and Second-Order Differential Equations in Pure mathematics.

3.1 Circular Model (CM)

Development of the CM is based on; Motion of a particle in a horizontal circle, Fourier transformation, and Least Square method of Multiple Regression. As explained in Konarasinghe W.G.S (2016); a real-valued function f(x) can be transformed into a series of trigonometric functions;

$$f_x = \sum_{k=1}^{n} a_k \cos k\theta + b_k \sin k\theta \qquad (1)$$

A particle *P*, which is moving in a horizontal circle of centre O and radius *a* is given in Figure 1. The ω is the angular speed of the particle;

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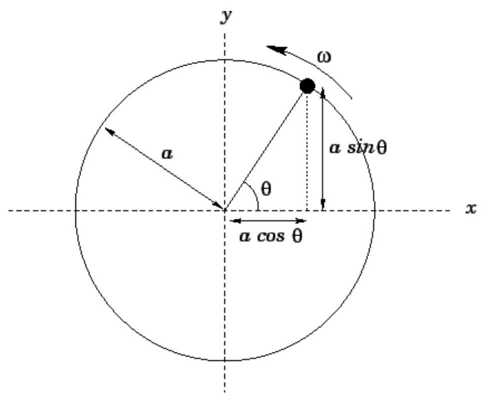

Figure 1: Motion of a particle in a horizontal circle

Angular speed is defined as the rate of change of the angle with respect to time. Then;

$$\omega = \frac{d\theta}{dt}$$

$$\int_0^{\theta} d\theta = \int_0^{t} \omega \, \mathrm{dt}$$

Hence, $\theta = \omega t$ (2)

Substitute (2) in (1); $f_x = \sum_{k=1}^{n} a_k \cos k\omega t + b_k \sin k\omega t$ (3)

Figure 2 and Figure 3 clearly show how to incorporate a particle in horizontal circular motion to trigonometric functions;

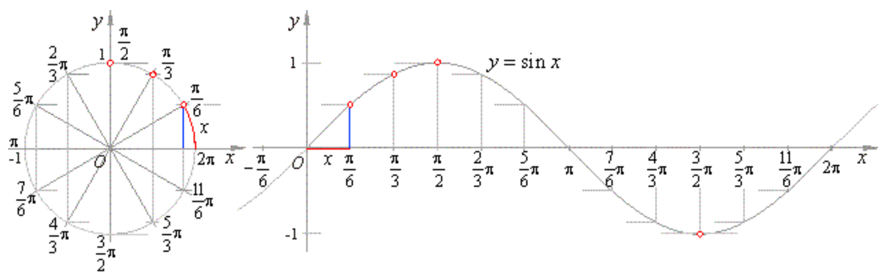

Figure 2: sine function and reference circle

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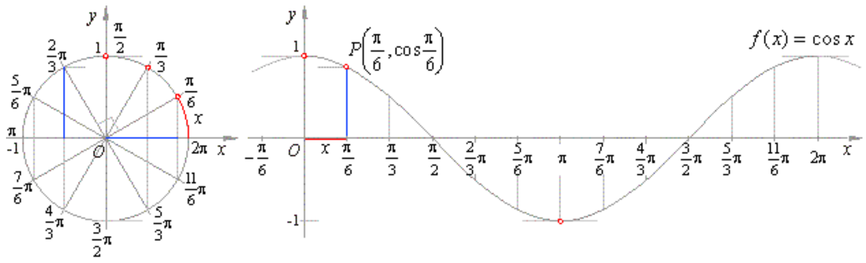

Figure 3: cosine function and reference circle

Reference to Figure 1; $\overrightarrow{op} = a(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$, where, a is the amplitude or wave height. A wave with constant amplitude is defined as a regular wave and a wave with variable amplitude is known as an irregular wave.

In circular motion, the time taken for one complete circle is known as the period of oscillation. At one complete circle θ=2π radians. Therefore, by (2), the time taken for one complete circle (T) is: $T = 2\pi / \omega$ (4)

If a time series follows a wave with f peaks in N observations, its period of oscillation can be given as;

$$T = \frac{\textit{total}\ \text{number of periods}}{\text{total number of peaks}} = \frac{N}{f} \tag{5}$$

Equating (4) and (5); $\dfrac{2\pi}{\omega} = \dfrac{N}{f}$

Then, $\omega = 2\pi \dfrac{f}{N}$

Hence (3) is modified as;

$$f_t = \sum_{k=1}^{n} [a_k \sin k(2\pi \frac{f}{N})t + b_k \cos k(2\pi \frac{f}{N})t] + \varepsilon_t \tag{6}$$

The model (6) was named as the "Circular Model".

3.2 Theories of Oscillations

Theories of oscillations considered in the study are as follows;

3.2.1 Simple Harmonic Motion

Consider a free oscillation of a particle on a straight line in a horizontal plane (Figure 4) and a motion of a particle on the reference circle (Figure 1);

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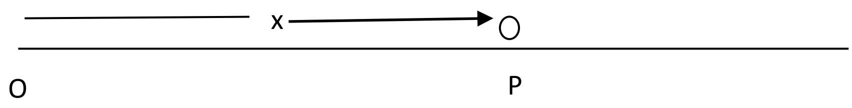

Figure 4

Using the Newton's law of motion;

$$F = ma$$
$$F = m\ddot{x} \tag{7}$$

$$x = a\cos\theta = a\cos\omega t$$
$$\frac{dx}{dt} = \dot{x} = -a\omega\sin\omega t \tag{8}$$
$$\frac{d^2x}{dt^2} = \ddot{x} = -a\omega^2\cos\omega t = -\omega^2 x$$

Hence $F = -m\omega^2 x$

When a particle is attached to an elastic string or spring and set into motion with no forces other than the tension/ thrust and gravity acting on it, the motion is known as Simple Harmonic Motion (SHM). The SHM can be described by Equations (7) (Susan et al. 2009);

$$\ddot{x} = -\omega^2 x \tag{9}$$

3.2.2 Damped Harmonic Motion

The simple harmonic oscillations have constant amplitudes, however, in practice, the amplitude of the oscillation decreases, and particle comes to rest with time due to friction, air resistance, etc. Hence the model (9) is refined, assuming a resistance *mcv* where *v* is the speed of the particle. The oscillations of this type are known as "Damped Harmonic Motion.

Using *F=ma*,

$$-m\omega^2 x - mcv = m\frac{d^2x}{dt^2}$$
$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + \omega^2 x = 0 \tag{10}$$

Equation (10) is solved to investigate the motion of the particle;

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$Let\ x = Ae^{\lambda t}$; A and λ are constants

Then; $\frac{dx}{dt} = A\lambda e^{\lambda t}; \frac{d^2x}{dt^2} = A\lambda^2 e^{\lambda t}$

Hence the Auxillary equation is; $A\lambda^2 e^{\lambda t} + cA\lambda e^{\lambda t} + \omega^2 Ae^{\lambda t} = 0$

$$Ae^{\lambda t}(\lambda^2 + k\lambda + \omega^2) = 0 \tag{11}$$

$Ae^{\lambda t} \neq 0$, the auxilary equation is $\lambda^2 + c\lambda + \omega^2 = 0$

Then, $\lambda = \frac{-c \pm \sqrt{(c^2 - 4\omega^2}}{2}$

The Auxiliary equation may have three solutions;

i. If $c^2 - 4\omega^2 > 0$, then two real and distinct roots. This is known as heavy damping or over damping.
ii. If $c^2 - 4\omega^2 = 0$, one real root. This is known as Critical damping
iii. If $c^2 - 4\omega^2 < 0$, one or two complex roots. This is known as Light damping or under damping where the oscillation with decreasing amplitude is observed.

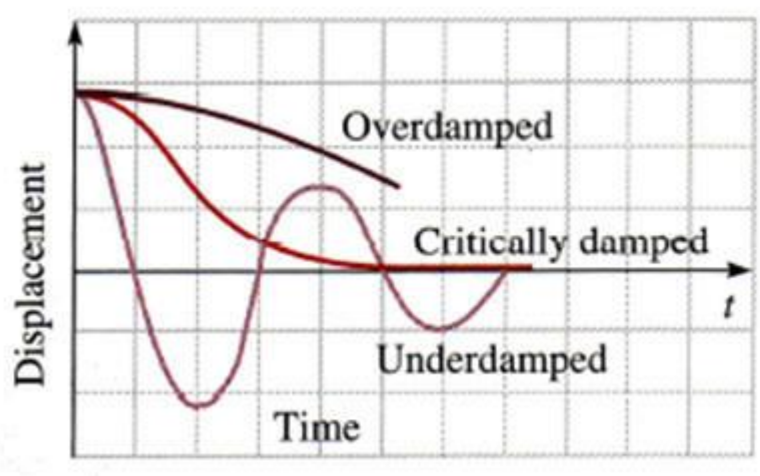

Figure 5: Damped Motions
Source; labman.phys.utk.edu

3.2.3 Forced Harmonic Motion

If the particle set in motion (Figure 4) is subject to a helping force, in addition to the restoring force, then the amplitude of oscillation increases as in Figure 6;

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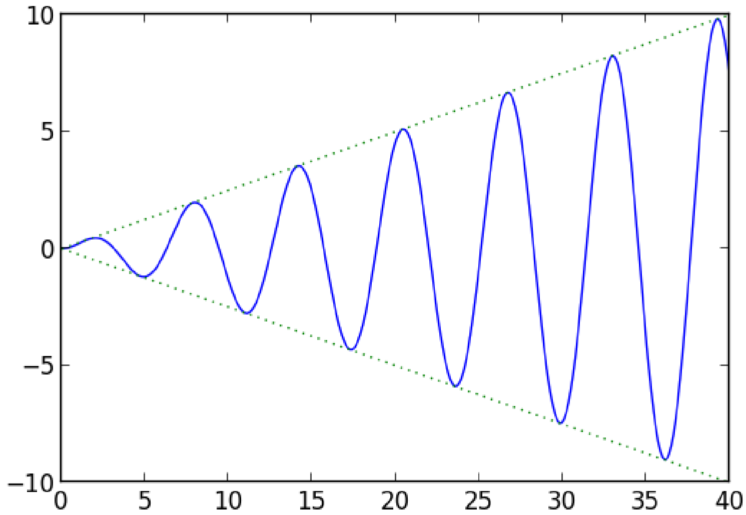

Figure 6: Forced Oscillation
Source: https://math-physics-

Assume a helping force *mcv* is applied on the particle in Figure 2, where *v* is the speed of the particle;

Using *F=ma*,

$$
\begin{aligned}
& -m\omega^2 x + mcv = m\frac{d^2x}{dt^2} \\
& \frac{d^2x}{dt^2} - c\frac{dx}{dt} + \omega^2 x = 0
\end{aligned}
\tag{12}
$$

Equation (12) is solved to investigate the motion of the particle;

$Let\ x = Ae^{\lambda t}$; A and λ are constants

Then; $\frac{dx}{dt} = A\lambda e^{\lambda t}; \frac{d^2x}{dt^2} = A\lambda^2 e^{\lambda t}$

Hence the Auxillary equation is; $A\lambda^2 e^{\lambda t} - cA\lambda e^{\lambda t} + \omega^2 Ae^{\lambda t} = 0$

$Ae^{\lambda t}(\lambda^2 + k\lambda + \omega^2) = 0$

$Ae^{\lambda t} \neq 0$, the auxilary equation is $\lambda^2 - c\lambda + \omega^2 = 0$

Then, $\lambda = \frac{c \pm \sqrt{(c^2 - 4\omega^2}}{2}$

If $c^2 - 4\omega^2 < 0$, the Auxiliary equation has complex roots, the particle will oscillates with increasing amplitude. This is known as Forced Oscillation (Bostock et. al. 1982).

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4. RESULTS

Results of the study consists two parts;

4.1. Development of the Damped Circular Model
4.2. Development of the Forced Circular Model

4.1 Development of the Damped Circular Model

Consider the general solution of equation (11) when $c^2 - 4\omega^2 < 0$;

$$\lambda = \frac{-c \pm \sqrt{(c^2 - 4\omega^2}}{2}$$

$$= \frac{-c}{2} \pm \frac{\sqrt{(c^2 - 4\omega^2}}{2}$$

Let $p = \frac{-c}{2}$ and $q = \frac{\sqrt{(c^2 - 4\omega^2}}{2}$

Then, $\lambda = p \pm iq$

Hence the general solution of (11);

$$x = K_1 e^{(p+iq)t} + K_2 e^{(p-iq)t}$$

$$= e^{pt}(K_1 e^{iqt} + K_2 e^{-iqt})$$

But, $e^{iqt} = \cos qt + i \sin qt,\ e^{-iqt} = \cos qt - i \sin qt$

Hence, $\text{x} = e^{pt}[(K_1(\cos qt + i \sin qt) + K_2(\cos qt - i \sin qt)$

$$x = e^{pt}[(K_1 + K_2)\cos qt + i(K_1 - K_2)\sin qt]$$

Let $A_1 = K_1 + K_2$ and $A_2 = i(K_1 - K_2)$

Then $x = e^{pt}(A_1 \cos qt + A_2 \sin qt)$

Where $p = \frac{-c}{2}$

$$x = e^{(-c/2)t}(A_1 \cos qt + A_2 \sin qt) ------------------------- (13)$$

As per the Fourier Transformation; $A_1 \cos qt + A_2 \sin qt = \sum_{k=1}^{n} [a_k \sin kq_k t + b_k \cos kq_k t]$

The Circular Model of Konarasinghe, W. G. S. (2016), modeled a random variable following an irregular wave like pattern by (6);

$$f_t = \sum_{k=1}^{n} [a_k \sin k(2\pi \frac{f}{N})t + b_k \cos k(2\pi \frac{f}{N})t] + \varepsilon_t$$

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Combining (6) and (13), a random variable (Y_t) with decreasing variation from the mean could be modeled as;

$$Y_t = e^{(-c/2)t} \sum_{k=1}^{n} [a_k \sin k(2\pi \frac{f}{N})t + b_k \cos k(2\pi \frac{f}{N})t] + \varepsilon_t \qquad (14)$$

Where; f is the number of peaks/ troughs of the time series of N observations.
The model in equation (14) is named as "Damped Circular Model (DCM)". Assumptions of the model are;

$t \geq 0, k \in R^{+}$

$\varepsilon_t \approx N(0, \sigma^2)$ and independent

Trigonometric series; $\sin k\omega t$ and $\cos k\omega t$ are independent; $\omega = 2\pi \frac{f}{N}$

4.2 Development of the Forced Circular Model

Consider the general solution of equation (12) when $c^2 - 4\omega^2 < 0$;

$$\lambda = \frac{c \pm \sqrt{(c^2 - 4\omega^2}}{2}$$

$$= \frac{c}{2} \pm \frac{\sqrt{(c^2 - 4\omega^2}}{2}$$

Let $p = \frac{c}{2}$ and $q = \frac{\sqrt{(c^2 - 4\omega^2}}{2}$

Then, $\lambda = p \pm iq$

Hence the general solution of (12);

$x = K_1 e^{(p+iq)t} + K_2 e^{(p-iq)t}$

$= e^{pt}(K_1 e^{iqt} + K_2 e^{-iqt})$

But, $e^{iqt} = \cos qt + i \sin qt, e^{-iqt} = \cos qt - i \sin qt$

Hence, $x = e^{pt}[(K_1(\cos qt + i \sin qt) + K_2(\cos qt - i \sin qt)$

$x = e^{pt}[(K_1 + K_2)\cos qt + i(K_1 - K_2)\sin qt]$

Let $A_1 = K_1 + K_2$ and $A_2 = i(K_1 - K_2)$

Then $x = e^{pt}(A_1 \cos qt + A_2 \sin qt)$

Where $p = \frac{c}{2}$

$x = e^{(c/2)t}(A_1 \cos qt + A_2 \sin qt)$ ------------------(15)

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As per the Fourier Transformation; $A_1 \cos qt + A_2 \sin qt = \sum_{k=1}^{n} [a_k \sin kq_k t + b_k \cos kq_k t]$

Combining (6) and (15), a random variable (Y_t) with increasing variation from the mean could be modeled as;

$$Y_t = e^{(c/2)t} \sum_{k=1}^{n} [a_k \sin k(2\pi \frac{f}{N})t + b_k \cos k(2\pi \frac{f}{N})t] + \varepsilon_t \qquad (16)$$

Where; *f* is the number of peaks/ troughs of the time series of *N* observations.

The model in equation (16) is named as "Forced Circular Model (FCM)". Assumptions of the model are;

$t \geq 0, k \in R^+$

$\varepsilon_t \approx N(0, \sigma^2)$ and independent

Trigonometric series; $\sin k\omega t$ and $\cos k\omega t$ are independent; $\omega = 2\pi \frac{f}{N}$

5. CONCLUSION AND RECOMMANDATIONS

Periodic motions and oscillatory motions are common in real life. Similar patterns are observed in time series data as well. It is observed that the time series data follow irregular wave like patterns with, constant amplitude; decreasing amplitude and increasing amplitudes. The stochastic models, ARIMA/ SARIMA; CM/SCM are capable in capturing wave like patterns with constant amplitudes, but not highly successful in modeling other two types. Hence the study was focused on model development for wave like patterns with increasing or decreasing amplitudes. Theories of Damped and Forced Oscillations, Solutions of Second Order Differential Equations and the Circular Model of Konarasinghe, W. G. S. (2016) were used in model development. Two new forecasting techniques named, Damped Circular Model (DCM) and Forced Circular Model (FCM) were the outcomes of the study.

It is recommended to test the DCM and FCM on real life data to access the forecasting ability of the models and compare them with ARIMA, SARIMA, CM and SCM.

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