

SCM and SARIMA on Forecasting COVID -19 Outbreak in Italy

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ABSTRACT

The COVID -19 originated from China in the year 2019 and grabbed more than 173,321, 279 human lives with a death total of more than 3,727,177 globally. Italy had been experienced with worst and the first European country severely hit by the COVID-19. At present daily infected cases shows a declining trend in Italy. Yet, the future is uncertain, it could rise again. The dynamic changes of the outbreak should be captured more often. Hence, the study has designed to forecast the daily infected cases of COVID -19 in Italy. The daily infected cases of COVID-19 of Italy for the period of 22nd January 2020 to 3rd June 2021 were obtained from the World Health Organization (WHO) database. The pattern recognition of the daily infected cases examined by time series plots and Auto Correlation Function (ACF). Based on the pattern the Auto-Regressive Integrated Moving Average (SARIMA) and the Sama Circular Model (SCM) were tested to forecast the daily infected cases in Italy. The model assumptions were tested by using the Anderson Darling test, ACF, and Ljung-Box Q (LBQ)-test. The forecasting ability of the models was assessed by three measurements of errors; Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE), and Mean Absolute Deviation (MAD) in both model fitting and verification processes. The results of the study revealed that the SARIMA and SCM were satisfied all validation criteria and the performance of the models were extremely high. It had been concluded that the SARIMA and SCM are suitable to forecast daily infected cases in Italy. It is strongly recommended to identify the hidden seasonal behaviors of other countries furthermore.

Keywords: COVID -19, Seasonal behavior, Daily Infected Cases

1. INTRODUCTION

1.1 Background of the Study

The COVID-19 first appeared in 2019 in China and grabbed the entire world exceeded 173,321,279 infected cases and 3, 727,177 death total. The term COIVD means Corona (CO), Virus (VI), Disease (D), and year 2019 (19), which is COVID-19 first appeared in 2019 (Sugiyanto & Muchammad, 2020). Novel coronavirus can cause pneumonia (Pan et al, 2020). Pneumonia is an inflammatory lung disease that is characterized by coughing, chest pain, fever, and difficulty breathing (Pan et al., 2020). The outbreak has made negative consequences to the economies and societies globally. Italy had been experienced with worst and the first European country severely hit by the COVID-19. At present they were capable to control the spread. World Health Organization (WHO) statistics have reported the first infected case from Italy on 31st January 2020. The total infected cases from Italy exceeded 4, 227,719, and more than 3,901,112 recovered. The total death reported 126, 415 or more up to now. At present daily infected cases shows a declining trend in Italy. It is a good sign to ensure the sleek function of the country.

1.2 Research Problem

Declining behavior has been observed in daily infected cases in Italy. Yet, the future is uncertain, it could rise again. Besides, there may be seasonal or cyclical behaviors of the outbreak. A study conducted by Konarasinghe, K.M.U.B (2021) has reported that there were many seasonal behaviors. Identifying such patterns of the pandemic in Italy will be most important to control the outbreak. Besides, the dynamic changes of the outbreak should be captured more often (Konarasinghe,K.M.U.B, 2021). The attention on seasonal behaviors of the outbreak was very less. Hence, forecasting the outbreak in Italy and capturing the seasonal behaviors is a timely requirement to control the outbreak of the pandemic.

1.3 Objective of the Study

The objective of the study is to forecast daily infected cases of COVID -19 in Italy.

1.4 Significance of the Study

The results of this study could be a guideline to develop various strategies related to health and medical care. Besides, it will be useful for the workout the logistic requirements and produce with minimum waste to control the outbreak in Italy (Konarasinghe,K.M.U.B.,2020-a). The results can be used to predict the patterns and speed of the outbreak in the country. It will be useful for decision-making for lockdown schedules, impose delivery systems, online activities, and implementation of mandates to control the unnecessary movements of the general public. The results of this study could be another guide for business developments. E-Commerce is one of the best practices during pandemic situations. Hence, the business should develop its business functions to gain maximum benefits based on the results of this study.

2. LITERATURE REVIEW

Modeling the spread of COVID-19 in Italy has followed several approaches including mathematical and statistical models. The review of the study was focused on forecasting outbreak of the pandemic in Italy and the applications of Sama Circular Model (SCM).

2.1 Studies Based on Modeling Outbreak in Italy

Dehesh et al (2020) have applied Auto Regressive Integrated Moving Average (ARIMA) to forecast number of daily confirmed COVID -19 cases in Italy, China, South Korea, Iran and Thailand. Logistic, Gompertz, Weibull, and Exponential Growth models applied by Attanayake et al (2020) to forecast daily cases in Sri Lanka, Italy, the United States, and Hebei Province of China. Antonin et al (2020) have applied Suspected, Exposed, Infected, Recovered and Locked –Down measures (SEIRL) model to predict the epidemic and analyze the effects of the imposed locked – down. Giuliani et al (2020) have applied susceptible (S), infected (I), diagnosed (D), ailing (A), recognized (R), threatened (T), healed (H) and extinct (E), termed SIDARTHE to model the pandemic in Italy. The same model have applied to predict the course of the pandemic by Giordano et al (2020). Suspected, Infected, and Recovered (SIR) model applied by Pizzuti et al (2020) to predict the spread of the pandemic in Italy. Kozyref (2020) have applied the same model to predict the number of beds occupied by the COVID-19 patients in Belgium, France, Italy, Switzerland and New York City and compare the results. Chu (2021) have predicted the daily and cumulative infected cases in Italy and Spain by applying the same model. In addition he applied log-linear regression model for the purpose. SIRD (Susceptible-Infected-Recovered-Died) model was applied by Ferrari et al. (2020) and Caccavo (2020) to predict the provincial cases in Italy and the Chinese and Italian outbreaks consecutively. Battineni et al (2020) have applied Susceptible (S) exposed (E), infected (I) and recovered (R) (SEIR) to model the outbreak in Italy.

2.2 Studies Based on the Applications of Sama Circular Model (SCM)

The Sama Circular Model (SCM) is a recently developed model to capture the trend, seasonal and cyclical variations of a time series. It is the improved version of the Circular Model (CM) of Konarasinghe, W.G.S.(2016). So far these models have not been applied in medical care and healthcare research. Yet Konarasinghe, W.G.S.(2020) has emphasis that the CM and SCM are applicable to model wave like patterns irrespective of the field of study. Hence the present study reviewed the applications of the CM and CSM in various fields of studies.

Konarasinghe, W.G.S. (2016) have tested the ARIMA and the Circular Model (CM) on forecasting Sri Lankan share market. Konarasinghe, W.G.S. (2018-a) have applied SCM, SARIMA and the Decomposition techniques on forecasting tourist arrivals to Sri Lanka. Konarasinghe, W.G.S. (2018-b) have applied SCM on forecasting two main stock market indices of Sri Lanka. Konarasinghe, W.G.S. (2019) used SCM and ARIMA models to forecast BSE Sensex Index of Bombay Stock Exchange (BSE) in India. The SCM and SARIMA were successfully applied to forecast foreign guest nights in Anuradhapura city in Sri Lanka (Konarasinghe, K.M.U.B, 2020-b). Konarasinghe, W.G.S.(2020) have compared the forecasting ability of SCM, ARIMA and Decomposition techniques in

forecasting female unemployment in Australia. As per the review, SCM outperformed Decomposition techniques, ARIMA/ SARIMA in various fields of studies.

High forecasting ability, following the similar pattern of actual vs. fittings and forecasting and capturing many seasonal and cyclical behaviors at once are the competencies of the SCM and CM. Most important property of the SCM is that it is able to separate the seasonal variations and cyclical variations without any effort (Konarasinghe, W.G.S, 2020). In contrast SARIMA, ARIMA and Decomposition techniques are unable to capture the cyclical behaviors. Konarasinghe, W.G.S. (2020) has recommended to apply the SCM in more real life situations of Meteorology; Agriculture; Healthcare and Biological Sciences; Business, Finance and more to get the benefit of it.

The SIR, SIRD and SIDARTHE were most common mathematical models applied to predict the pandemic in Italy. The SEIRL, SIRD and SEIR were other mathematical models applied for the purpose. Besides, ARIMA, Logistic, Gompertz, Weibull, Exponential Growth and Log-linear regression models were the statistical models applied by the researchers. The researcher's concern on model verification was less and the measurements of the forecasting ability was doubtful in several models. The seasonal patterns of the daily infected cases were not identified properly.

3. METHODOLOGY

The daily infected cases of COVID-19 in Italy for the period of 22nd January 2020 to 3rd June 2021 were obtained from the World Health Organization (WHO) database. Pattern recognition of the daily infected cases paves the path for the selection of the most suitable model (Konarasinghe, K.M.U.B. 2016-a; 2016-b) and (Konarasinghe, W.G.S., & Abeynayake, 2014). It gives an insight into the various behaviors of trends, seasonal variations, cyclical variations, and heavy and minor volatility within the precise period of daily infected cases in Italy (Konarasinghe, W.G.S. & Abeynayake, 2014). Hence, time series plot and Auto Correlation Function (ACF), were used for the aim, as done by Konarasinghe, W.G.S., & Abeynayake (2014). Based on the pattern of the data series, Seasonal Auto Regressive Integrated Moving Average (SARIMA) and Sama Circular Model (SCM) were tested to forecast the daily infected cases in Italy. The model assumptions were tested by using the Anderson Darling test, ACF, and Ljung-Box Q (LBQ)-test (Konarasinghe, W.G.S. et al, 2015). Three measurements of errors used to assess the forecasting ability. They are; Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE) and Mean Absolute Deviation (MAD) in both model fitting and verification process, as per Konarasinghe, K.M.U.B. (2015- a ;2015-b).

3.1 Seasonal Auto Regressive Integrated Moving Average (SARIMA)

ARIMA modeling can be used to model many different time series, with or without trend or seasonal components, and to provide forecasts. The model as follows;

An ARIMA model is given by:

$$\phi(B)(1-B)^d y_t = \theta(B)\varepsilon_t$$

$$\text{Where; } \phi(B) = 1 - \phi_1 B - \phi_2 B^2 \dots \phi_p B^p$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 \dots \theta_q B^q \quad (1)$$

ε_t = Error term

D = Differencing term

B = Backshift operator ($B^a Y_t = Y_{t-a}$)

Analogous to the simple SARIMA parameters, these are:

Seasonal autoregressive - (Ps)

Seasonal differencing - (Ds)

Seasonal moving average parameters - (Qs)

Seasonal models are summarized as ARIMA (p, d, q) (P, D, Q)_s

Number of periods per season - S

$$(1 - \phi_1 B)(1 - \phi_1 B^s)(1 - B)(1 - B^s)Y_t = (1 - \theta_1 B)(1 - \theta_1 B^s)\varepsilon_t \quad (2)$$

3.2 Circular Model (CM) and Sama Circular Model (SCM)

The development of the CM was based on; Fourier Transformation, the theory of Uniform Circular motion and Multiple Regression Analysis (Konarasinghe, W.G.S., 2016). The SCM is the improved version of the CM (Konarasinghe, W.G.S., 2018-b).

3.2.1 Circular Model (CM)

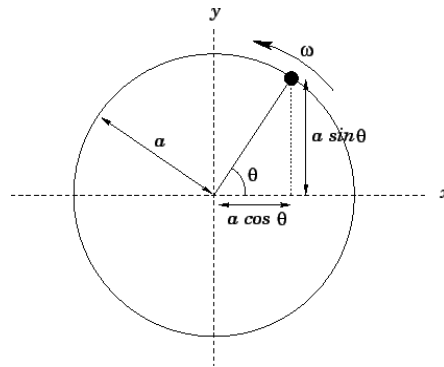
As explained in Konarasinghe (2016), a discrete version of the Fourier transformation for a function $f(x)$ is written as:

$$f_x = \sum_{k=1}^n a_k \cos k\theta + b_k \sin k\theta \quad (3)$$

Where a_k and b_k are amplitudes, k is the harmonic of oscillation.

The Fourier transformation is incorporated into a uniform circular motion of a particle in a horizontal circle and basic trigonometric ratios (Konarasinghe, W.G.S., 2016). A particle P , which is moving in a horizontal circle of center O and radius a is given in Figure 1. The ω is the angular speed of the particle;

Figure 1: Motion of a particle in a horizontal circle



Angular speed is defined as the rate of change of the angle with respect to time. Then;

$$\omega = \frac{d\theta}{dt}$$

$$\int_0^\theta d\theta = \int_0^t \omega dt$$

Hence, $\theta = \omega t$ (4)

Substitute (4) in (3); $f_x = \sum_{k=1}^n a_k \cos k\omega t + b_k \sin k\omega t$ (5)

At one complete circle $\theta=2\pi$ radians. Therefore, the time taken for one complete circle (T) is given by: $T = 2\pi / \omega$

Figure 2 and Figure 3 clearly show how to incorporate a particle in a horizontal circular motion to trigonometric functions;

Figure 2: sine function and reference circle

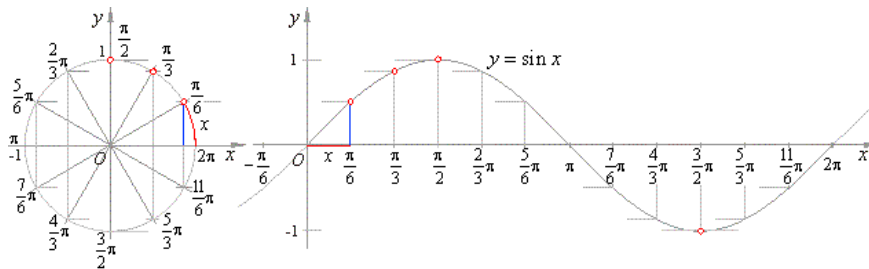
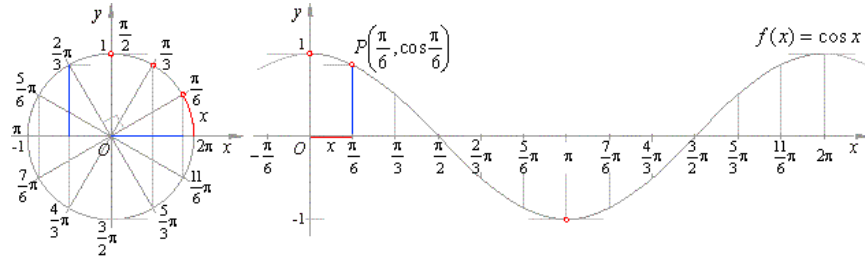


Figure 3: cosine function and reference circle



Reference to Figure (1) ; $\vec{op} = a(\cos \theta \mathbf{i} + \sin \theta \mathbf{j})$, where, a is the amplitude or wave height. A wave with constant amplitude is defined as a regular wave and a wave with variable amplitude is known as an irregular wave. In a circular motion, the time taken for one complete circle is known as the period of oscillation.

In other words, the period of oscillation is equal to the time between two peaks or troughs of sine or cosine function. If a time series follows a wave with f peaks in N observations, its period of oscillation can be given as;

$$T = \frac{\text{total number of periods}}{\text{total number of peaks}} = \frac{N}{f} \quad (6)$$

$$\text{Hence, } \omega = 2\pi \frac{f}{N} \quad (7)$$

Therefore, a variable Y_t , with an irregular wave pattern was modeled as;

$$Y_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (8)$$

The model (8) was named as ‘‘Circular Model’’.

Model assumptions of the CM are; the series Y_t is trend-free; trigonometric series, k is a positive real number, $\sin k\omega t$ and $\cos k\omega t$ are independent; residuals are Normally distributed and independent.

3.2.2 Sama Circular Model (SCM)

A limitation of the CM is that it is not applicable for a series with a trend. Konarasinghe, W.G.S. (2018-a; 2018-b) suggests the method of differencing to mitigate the limitation of the CM. In usual notation, differencing series of Y_t are as follows;

$$\text{First differenced series: } Y_t' = Y_t - Y_{t-1} = (1 - B)Y_t \quad (9)$$

Second differenced series:

$$Y_t'' = Y_t' - Y_{t-1}' = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2}) = Y_t - 2Y_{t-1} + Y_{t-2} = (1 - B)^2 Y_t \quad (10)$$

$$\text{Similarly, } d^{\text{th}} \text{ order difference is, } Y_t^d = (1 - B)^d Y_t \quad (11)$$

Where, B is the Back Shift operator; $BY_t = Y_{t-1}$.

Assume Y_t^d is trend-free. Let, $Y_t^d = X_t$. Then X_t could be modeled as;

$$X_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (12)$$

$$\text{Hence; } (1 - B)^d Y_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (13)$$

the model (13); improved Circular Model, is named as ‘‘Sama Circular Model (SCM)’’.

4. RESULTS

The analysis contains two main parts:

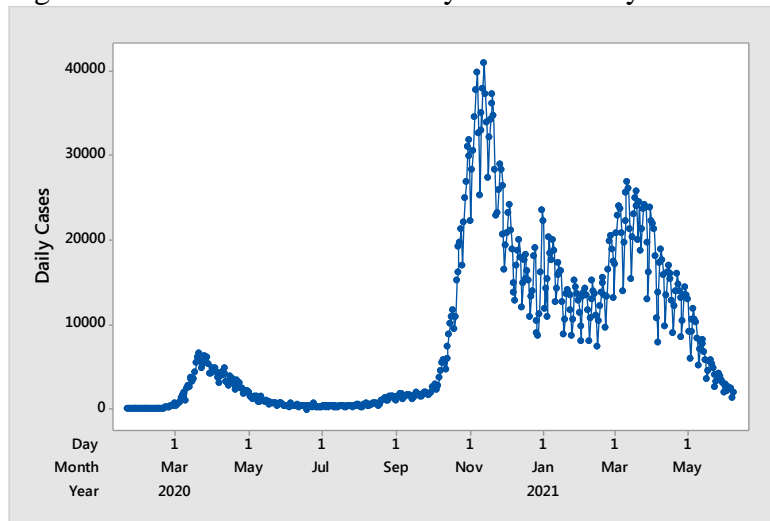
- 4.1 Pattern recognition of daily infected cases in Italy.
- 4.2 Forecasting daily infected cases in Italy.

Initially, pattern recognition of the data series of Italy was examined, then SARIMA and SCM models tested for forecasting. Log transformed data were used for the analysis.

4.1 Pattern Recognition of Daily Infected Cases in Italy.

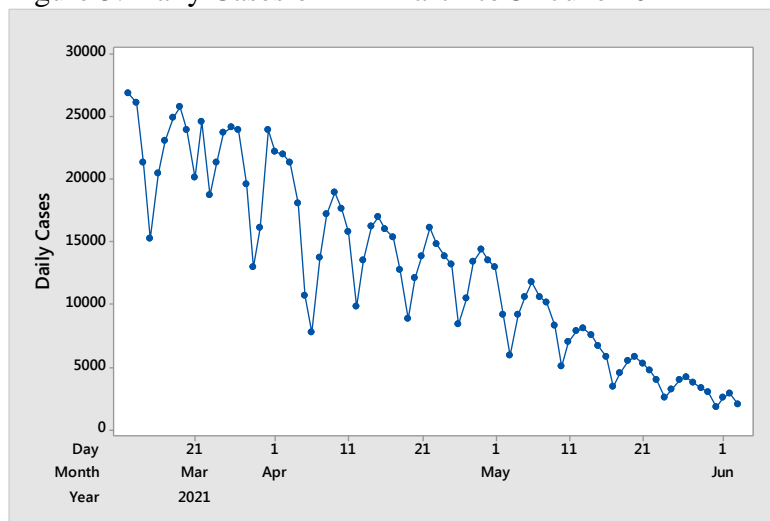
Time series plot of daily infected cases of Italy for the period of 22nd January 2020 to 3rd June 2021 (Figure 4). The first confirmed case reported from Italy on 31st January 2020. There was an exponential growth of daily infected cases for the period of 16th October to 13th November 2020. Declining trend was observed for the period of 14th November 2020 to 16th February 2021. Once again there was a growth for the period of 17th February to 12th March 2021. Declining trend was observed after 12th March 2021.

Figure 4: Time Series Plot of Daily Cases in Italy



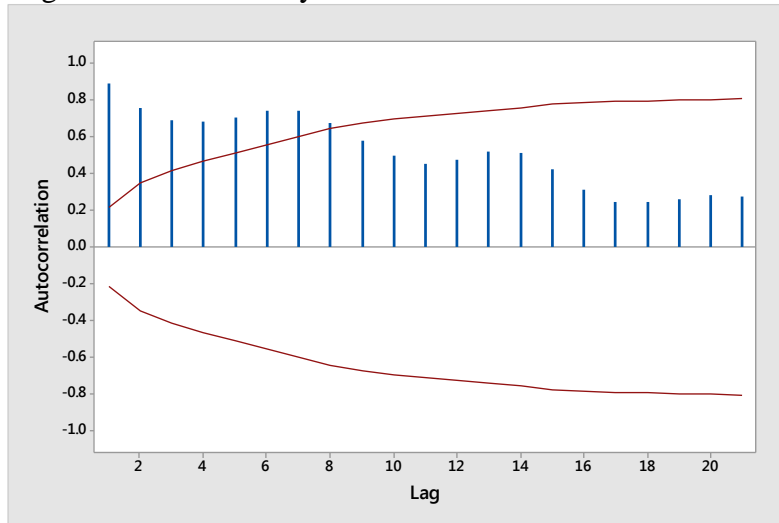
Hence, the data set for the period of 12th March 2021 to 3rd June 2021 was used to forecast daily infected cases in Italy. The pattern of the daily infected cases for the selected period was further examined. Figure 5 is that the time series plot of confirmed cases for the period of 12th March to 3rd June 2021.

Figure 5: Daily Cases of 12th March to 3th June 2021



There is a declining trend of daily infected cases with fluctuations. After 17th May 2021, the volatility is less. The behavior of Figure 5 suggested the linear trend of the series. The ACF of the series is shown in Figure 6.

Figure 6: ACF of Daily Cases



The ACF of Figure 6 shows several seasonal behaviors with a declining trend along with many significant lags of the daily infected cases in Italy. There could be several seasons including the lengths of 7 days or more or less. Besides, the series confirmed the weak stationary of the series. The observed pattern of the series suggested SARIMA model is suitable to capture the seasonal behavior. The SCM is another technique to capture seasonal behaviors with the trend and it can identify many seasonal behaviors at once. Hence, the study selected SARIMA and SCM to forecast daily infected cases in Italy.

Figure 7: First Difference Series of Daily Cases

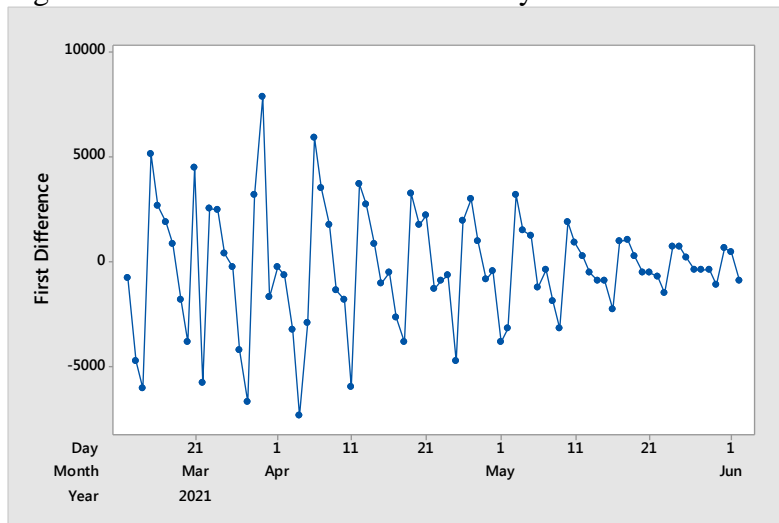


Figure 7 is the time series plot of the first difference series of the daily infected cases. The first difference series is used to examine the wave pattern of the series and apply SCM (Konarasinghe, W.G.S., 2018-a). The series shows a wave-like pattern with high

volatility at the beginning and lower at the end. Figure 7 is another evidence to select SCM to forecast daily infected cases in Italy.

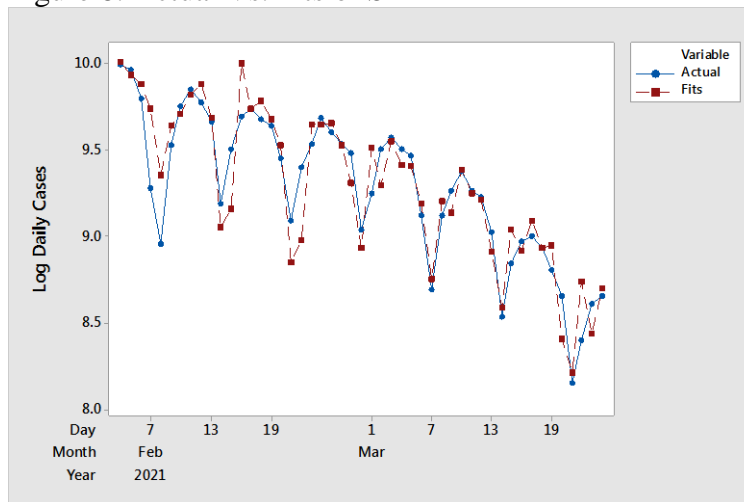
4.2 Forecasting Daily Infected Cases in Italy

Several SARIMA models were tested on the series. The SCM was run on 28 trigonometric series. The results of the Table 1 revealed that there were 4 significant trigonometric series out of 28. They are; $\sin \omega t$, $\sin 4.5\omega t$, $\sin 5.5\omega t$ and $\cos 5.5\omega t$. The Anderson Darling test, LBQ test, and the ACF have confirmed the normality and the independence of the residuals of ARIMA (1,0,0)(1,3,2)₇ and SCM. The results of the model validation criterion have shown in Table 1. The measurements of errors are very low under the fitting and verifications of both the ARIMA (1,0,0)(1,3,2)₇ and SCM. The summaries of the SARIMA and SCM are given in Table 1;

Table 1: Summary of Model Fittings and Verifications of SARIMA and SCM

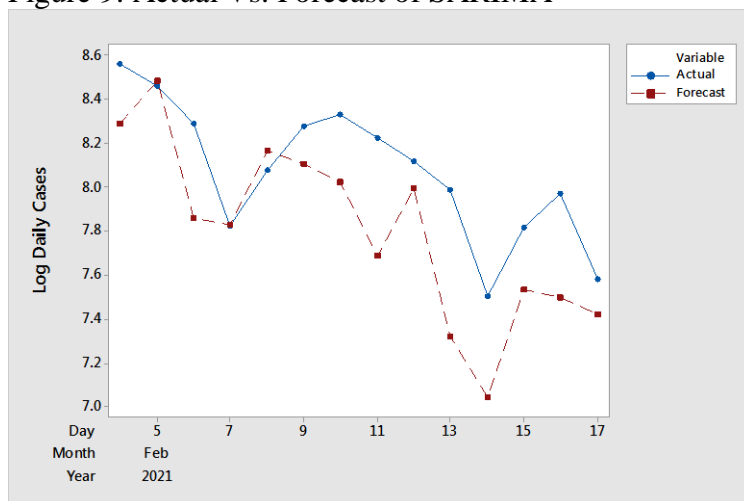
Model	Model Fitting		Model Verification	
ARIMA(1,0,0)(1,3,2) ₇	MAPE	1.3383	MAPE	3.5755
	MSE	0.0288	MSE	0.1212
	MAD	0.1228	MAD	0.2875
	Normality	P= 0.064		
	Independence of Residuals	Yes		
$Y_t = Y_{t-1} + 0.0812 \sin \omega t$ $+ 0.0969 \sin 4.5\omega t$ $+ 0.1391 \sin 5.5\omega t - 0.1371 \cos 5.5\omega t$	MAPE	1.5784	MAPE	3.5148
	MSE	0.0335	MSE	0.1055
	MAD	0.1478	MAD	0.2806
	Normality	P=0.327		
	Independence of Residuals	Yes		

Figure 8: Actual Vs. Fits of SARIMA



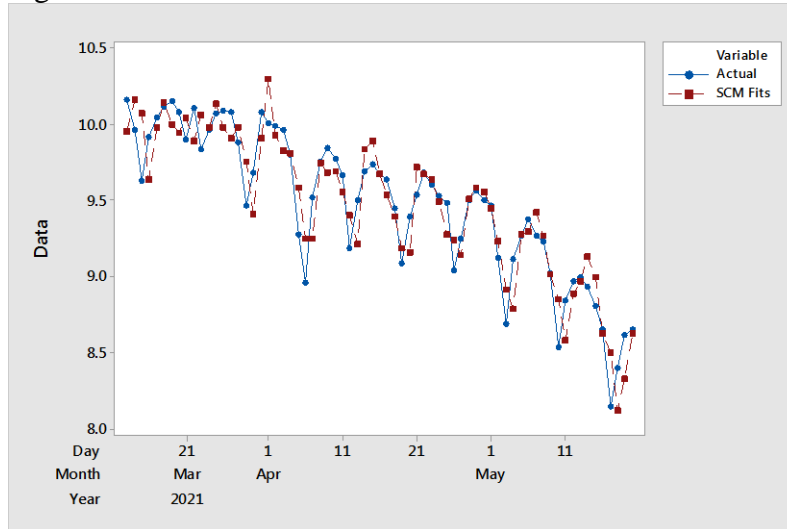
The actual and fits of the ARIMA (1,0,0)(1,3,2)₇ are in Figure 8. Actual and fits of the model are closer to each other and follow similar patterns of each other.

Figure 9: Actual Vs. Forecast of SARIMA



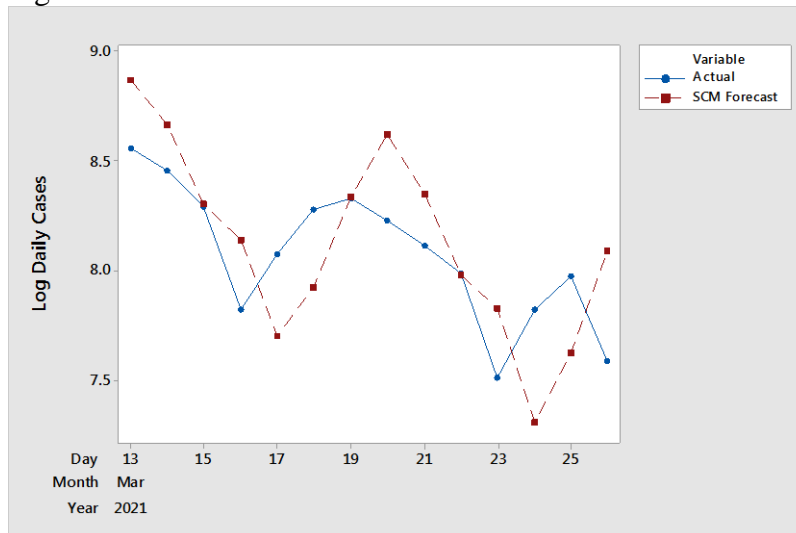
The underestimation behavior observed in daily infected cases in Figure 9. But, the actual Vs. forecast of ARIMA (1,0,0)(1,3,2)₇ shows the least deviation. The results of Table 1 and the behavior of Figures 8 and 9 are confirmed that the ARIMA (1,0,0)(1,3,2)₇ is suitable to forecast daily infected cases in Italy. Figure 10 is the actual vs. fits of SCM. The fits of the daily infected cases in Italy follow a similar pattern to the actual series.

Figure 10: Actual Vs Fits of SCM



The deviations also very less between the actuals and fits of SCM. The behavior of SCM is similar to the behavior of ARIMA (1,0,0)(1,3,2)₇ under the fitting process. Figure 11 is the actual vs. forecast of SCM. The deviations between actual and forecast are very less.

Figure 11: Actual Vs. Forecast of SCM

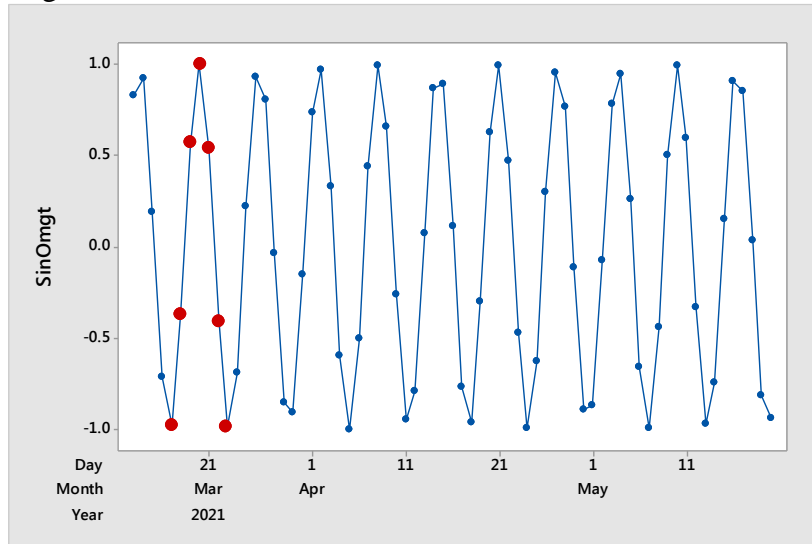


The forecasting ability and the behaviors of the actual vs. fits, and forecast of SCM confirm that the SCM is suitable to forecast daily infected cases in Italy. The model of SCM given below;

$$Y_t = Y_{t-1} + 0.0812 \sin \omega t + 0.0969 \sin 4.5 \omega t + 0.1391 \sin 5.5 \omega t - 0.1371 \cos 5.5 \omega t \quad (14)$$

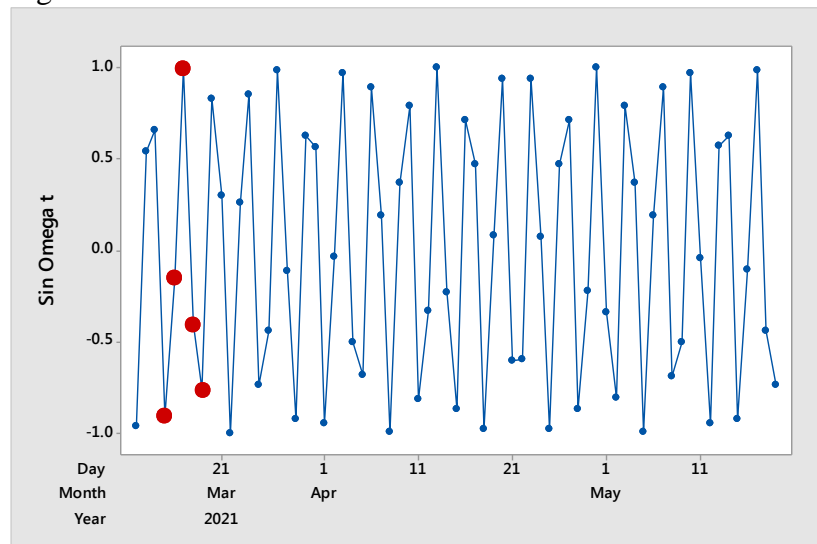
There were 4 significant trigonometric series in the best fitting SCM (14). They are; $\sin \omega t$, $\sin 4.5\omega t$, $\sin 5.5\omega t$ and $\cos 5.5\omega t$ shown in Figures 12, 13, 14 and 15. Figure 12 is the plots of $\sin \omega t$.

Figure 12: Plot $\sin \omega t$



The wave length of Figure 12 is 7 days. That means daily infected cases in Italy rise and fall in 7-day intervals.

Figure 13: Plot $\sin 4.5\omega t$



The wave length of Figure 13 of $\sin 4.5\omega t$, shows 5 days. Daily infected cases in Italy rise and fall in 5 days shown in Figure 13.

Figure 14: Plot $\sin 5.5\omega t$

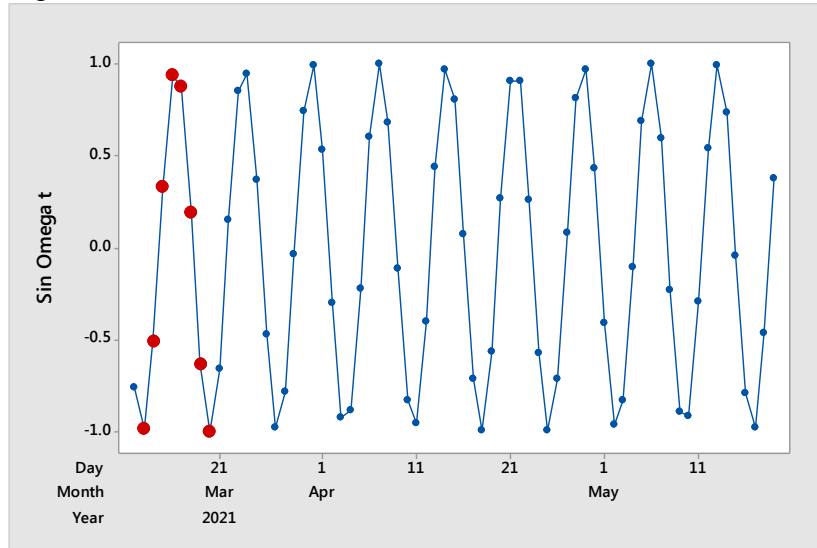
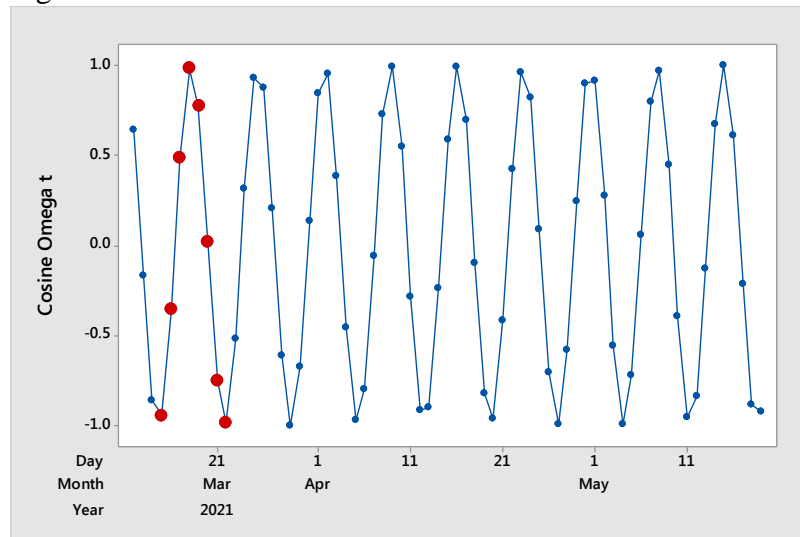


Figure 14 is the plot of $\sin 5.5\omega t$. The wave length of Figure 14 is 8 days. Daily infected cases in Italy rise and fall in 8 days shown in Figure 14.

Figure 15: Plot $\cos 5.5\omega t$



The same wave length 8 days in Figure 15 of $\cos 5.5\omega t$. The rise and fall of daily infected cases in Italy in 8 days.

5. CONCLUSION AND RECOMMENDATION

It is concluded that ARIMA(1,0,0) (1.3.2)₇ and SCM are the best-suited models in forecasting daily infected cases in Italy. The performance of both models was extremely high, but, SCM is the most powerful model than SARIMA in this situation. The deviations of SCM were less in verifications than SARIMA. The SARIMA captured one seasonal pattern at a time, whereas, SCM was capable of capturing 4 seasonal behaviors at a time with minimum effort.

The behaviors of the daily infected cases in Italy show rise and fall. The rise and fall exist in 5, 7, and 8 days. Capturing the seasonal changes of the rise and fall of daily infected cases could be one of the latest and useful findings in COVID – 19 research. Identification of the seasonal behavior of the daily infected cases will be one of the most useful guidelines to control the outbreak of the pandemic in Italy. It will be better guidance to control the movements and other activities of the general public. Lockdown schedules and other movement restrictions can be imposed methodically without causing more difficulties to society.

There were few medical care treatments introduced to protect from COVID-19. But, medical care constitutes only 10% to 20% of health outcomes approximately (AIM, 2019). That is not sufficient for complete prevention especially from a virus. The remaining 80% to 90% incorporates social factors including several lifestyles. Working styles, smoking, consuming too much alcohol, food habits, consuming unhealthy food (short eats and other fast food), personal hygiene, etc (AIM, 2019). The general public should be educated by authorities through media to follow healthcare practices and avoid unhealthy activities to prevent COVID-19. It could be a contribution to avoid the outbreak of the pandemic.

Self-discipline is one of the most important traits to be developed by the general public. The authorities should impose new mandates to ensure the self-discipline and the quality with safety standards of health care products. The quarantine procedures should be followed strictly and monitored their health condition for a certain period of time. The government should concentrate to produce food and beverages to improve the immunity of the human body and restrict unhealthy food and beverages. Improving immunity is another factor to prevent the COVID- 19.

According to Lowen, (2021), there is a crippling in Italy's birth rates since 1861. Now it's declining every year. The countries births were heavily outnumbered by deaths, according to the Lowen, (2021). This is one of the crucial situations has experienced by the nation of Italy. The present pandemic situation could be another negative impact on birth rates and increased the number of death rates. Due to the prevailing situation, the authorities should concentrate more on health care systems to protect the human resource and increase the birth rates and minimize the death rates in the country. This pandemic could be another reason to increase the present death rate. Then the outbreak of the pandemic should be control.

Hence, the authorities should identify the speed and the pattern of the outbreak with the help of the results of this study.

There could be several seasonal behaviors available in other COVID -19 datasets. Capturing the seasonal effects of daily infected and recovered cases of other countries will be more important to examine. Hence it is strongly recommended to identify the hidden seasonal behaviors of other countries furthermore.

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