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Frobenius Method on a System of Coupled Differential Equations with Stochastic Variables Arising in Financial Markets

I.U. Amadi[1*], I. Davies[2], A.O. Nwaoburu[3]
Department of Mathematics/ Statistics
Captain Elechi Amadi Polytechnic, Rumuola Port Harcourt, Nigeria[1]
Department of Mathematics
Rivers State University Nkpolu Oroworokwo,
Port Harcourt, Nigeria [2, 3]

innocent.amadi@portharcourtpoly.edu.ng[1*]
davies.iyai@ust.edu.ng [2]
nwaoburu.adols@ust.edu.ng[3]

ABSTRACT

This paper considered Black-Scholes partial differential equation which was later reduced to system of coupled differential equations with some stochastic financial market variables in the model. These problems were solved analytical by adopting the Ferobenius method of series solution and two different investment solutions were obtained accordingly. The necessary conditions were achieved which govern asset price return rates through additive effects, additive inverse effects, and asset growth-rates of assets parameter respectively .Therefore, the impressions on the value of asset prices of investors in financial market were analyzed using graphical results of stock variables. Finally, the influences of the relevant parameters of stochastic variables were effectively discussed all in this paper.

Keywords**:** Stock Prices, Asset Value, Series solution, Stochastic analysis and Return rates

1. INTRODUCTION

1.1 Background of the Study

Financial markets is an organize system or place that gives opportunity for buyers and sellers to trade on financial instruments such as bond, equities, currencies and financial derivatives etc. Generally, those who needs capital and those who have capital to invest are been facilitated by financial markets such that investors regains their financial assets. Therefore ,financial assets is aimed towards bringing good returns to the investors over time in both long and short term business plans. Corporate bodies, individual or group of persons accumulate wealth to make a comfortable living in their families through financial assets. Though, in financial markets of an investment; rate of return is expected to cover-up unforeseen risks of the investments on cash flows acquired by the investors as cash dividends over the period of trading. In view of all this, the investor must take considerable account on the safety of the return rates of the investment to avoid some unforeseen circumstances which may have direct or indirect effects on the investments during the season. Nevertheless, capital investment enhances human wants effectively that is why an investor has to be prudent in every ramifications to avoid unnecessary loses. So, for proper actualization of an investment there should be diversification of asset allocation in order to improve on the expected rate of returns to cover-up some loop-holes and to reduce overall portfolio risk in the investment. The above concepts are achievable through realistic mathematical models. There are various mathematical models to serve the purpose of asset pricing; indeed all of them are highly successful under different conditions. For instance, a mathematical model to be developed will amount to an accurate analytical solution of Ferobenius method which can realistically stimulate stock variables or quantities for better results; as it arises in financial markets that is why a variable coefficient problem of 2^{nd} order differential equation with stochastic variables needs to be captured in the model. Hence, a differential equation is used to analyze the inter-relationships that exist between different components, then coupling into a single system such that it will continue to be independent of each order, Davies (2006**).** In mathematical modeling of engineering and sciences differential equations **is** one of the most frequently used tools for proper decision making.

1.2 Research Problem

In normal circumstances the analytical solution of variable coefficient problem of Ferobenius method is non-trivial one in deed, it is challenging when imposed with some stochastic and financial market variables are in the model. The greatest of it is when the analysis of the problem is being required. So the ability to appropriately understand the analytical solution to suit the explanations of financial market is not an easy one to comprehend as to apply adequately. Hence, this paper investigated a coupled system of investment equations to realistically assess the value of asset growth-rates when it follows additive effects.

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1.3 Objective of the Study

To propose an investment equation that will determine the value of assets and its return rates through analytical solutions.

1.4 Significance of the Study

This paper focused on two system of second order ordinary differential equations with stochastic and growth-rate parameters in the model whose rate of returns is assumed to follow: additive and additive inverse effects; all taking quadratic functions over the period of time. These investment equations were solved analytically by adopting series solution of Ferobenius method to obtain two complimentary solutions of the homogenous side of the equations and non-homogenous part with the method of undetermined coefficient respectively. To authenticate the rationality of the analytical solutions, the value of assets and relevant parameters in the model were assessed using detailed conditions of predicting stock returns and asset valuations; and the effects of rate of returns, asset valuation and stock volatility and growth-rates of asset were demonstrated graphically. This paper compliments the work of Amadi et al. (2022), by incorporating additive effects and additive inverse effects parameters and solving it as a coupled system of second order ordinary differential equations. To this end, this paper presents the profiles of asset value when it follows additive effects, additive inverse effects, growth rate of assets when its return rate is additive inverse effects which informs investors, traders, opinion leaders, economist and government for the purpose of decision making.

2. LITERATURE REVIEW

So many scholars have written extensively on differential equations whose results have been obtained in diverse ways. For instance Davies et al. (2019) considered differential equation model for unstable nature of stock market forces. In their result they explored the properties of fundamental matrix solution to determine stock price changes. In the work of Nwobi and Azor (2022) differential equation were developed with stochastic parameter in the model, the problem were solved analytical to obtain equilibrium prices in variation of market price changes. On the contrary Tian-Quan and Tao (2010) examined stock prices of share in A and H stock markets using a set of simultaneous differential equations; where iteration was applied and results obtained accordingly. In view that Amadi et al. (2017) considered two competing growth – rates on stock market prices. In their work differential equation was applied to obtain results based on minimum variance criteria. In the same vain Durujave and Uzoma (2020) considered a problem of stock market analysis, with two competing growth-rates and carrying capacity. They applied differential equation and results obtained based on minimum variance. More so, in the work of Elzaki (2011) solved variable coefficient problems of differential equation were solved analytical by the means of Elzaki transform and result obtained for the purpose of prediction. In another dimension Amadi et al. (2022) studied system of four differential equations where stochastic parameter and stock return rates was incorporated in the model; results showed an asset price dynamics to obtain stock of

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rate of returns. Osu and Amadi (2023) solved an investment equation to establish the behavior of stock return rates and asset price valuation when it follows multiplicative, additive effects respectively.

In all, a significant number of scholars has written extensively on financial market investment dynamics; see for example, Ugbebor et al.(2001**),** Edeki et al. (2015), Offomata et al. (2017),Charlene et al. (2017),Kasozi (2022), Kasumo et al.(2020), Osu et al. (2022), Wokoma et al.(2022),George and Kenneth (2019), Ofomata (2017) , Osu and Amadi (2022) and Aggarwal et al.(2018) etc .

However, from the literature Amadi et al.(2022) , Osu and Amadi (2023) etc considered various forms of asset valuations but did not consider asset growth-rates when its rate of returns follows additive effects as this widens the scope of applicability in this dynamic area of mathematical finance.

3. METHODOLOGY

Here we present few rudimentary preliminaries touching the dynamics of this study, therefore we have the following:

A Stochastic Differential Equation (SDE) is integration of differential equation with stochastic terms. So, in considering the Geometric Brownian Motion (GBM) which govern price dynamics of a non-dividend paying stock as:

$$dS(t) = \mu S(t)dt + \sigma S(t)dz(t), \tag{1}$$

Where S denotes the asset value, μ is the stock rate of return (drift) which is also known as the average rate of the growth of asset price and σ denotes the volatility otherwise called standard deviation of the returns. The $dz(t)$ is a Brownian motion or Wiener process which is defined on probability space $(\Omega, \mathrm{F}, \wp)$. However, stock price follows the Ito's process and the drift rate is stated as follows:

$$\mu = \left(\frac{\partial f}{\partial S_t} a_t + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S_T^2} b_t^2 \right), \tag{2}$$

$$\sigma^2 = \frac{\partial^2 f}{\partial S_t^2} b_t^2. \tag{3}$$

However, Ito's process is a stochastic process $\{X_t, t \geq 0\}$ known as Ito's process which follows:

$$X_t = X_0 + \int_0^t (\mathrm{t}, \varpi) d\tau + \int_0^t b(t, \varpi) dz_t. \tag{4}$$

Where $a(t,\varpi)$ and $b(t,\varpi)$ are adapted random functions? Thus, from initial stock price S_0 at time 0 adopting Ito's theorem gives the solution below:

$$S(t) = S_0 \exp\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma dz(t). \tag{5}$$

Where dz is a standard Brownian motion.

From (1) Black-Scholes Partial Differential Equation (PDE) is given as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0. \tag{6}$$

Following the method of Osu (2010).

$$dS(t) = \hat{\alpha} S(t)dt + \sigma S(t)dW(t), \hat{\alpha} = \alpha + \lambda. \tag{7}$$

Where λ is the market price of risk. The equation governing stock option is backward Black-Scholes partial differential equation given in one variable. In Osu (2010) such a mathematical model were studied in order to determine stock price fluctuations.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (\alpha - \lambda)S\frac{\partial V}{\partial S} - ru = 0. \tag{8}$$

Assume $\lambda = 0$ (8) gives:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \alpha S\frac{\partial V}{\partial S} - rV = -\frac{\partial V}{\partial t}, \tag{9}$$

From (9) $V \neq V(t)$ hence V is a function of S alone and we attained at the succeeding 2nd order nonhomogeneous differential equation by also crucial the index price of the form: $-\varphi - GR_t$ which substitutes right hand side of (9) and in understanding of the combined intrinsic value of stock , hereafter it encompasses the volatility of the underlying asset through the trading days. Moreover, risk factors S describe any kind of risk and uncertainty present in the financial market such as stock prices, interest rate, strike price etc. So linking the above factors and carrying out some simplification gives a complete 2^{nd} non-homogenous ordinary differential equation below:

$$S\frac{d^2V}{dS^2} + \frac{dV}{dS} - S\sigma^2 V = -\varphi - GR_t. \tag{10}$$

Where φ represents constant and GR_t is growth rate of the underlying asset?

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3.1 Problem Formulation of Investment Equation

Considering asset values and its rate of return which in grows explicitly: additive effects series and additive inverse effects series respectively at time t and they are quadratic, which is considered by fluctuations due to some environmental special effects. So, we consider companies with different portfolio of investments selected to be represented as S_2 and S_5 be defined in the system of 2^{nd} order differential equations. Henceforth the rate of returns is well-defined as follows:

$$\left.\begin{array}{l} R_t := \left(\lambda_1+\lambda_2\right)^2, \ldots \text{where } t=1,2\ldots \\ R_t := \left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^2, \ldots \text{where } t=1,2.. \\ . \end{array}\right\}. \tag{11}$$

Using (10) in (9) independently gives the following investment equations of two coupled differential equation:

$$S_2\frac{d^2V_{2(t)}}{dS_2{}^2}+\frac{dV_{2(t)}}{dS_2}-(\lambda_1+\lambda_2)^2S_2\sigma^2V_{2(t)}=0. \tag{12}$$

$$S_5\frac{d^2V_{5(t)}}{dS_5{}^2}+\frac{dV_{5(t)}}{dS_5}-\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2S_5\sigma^2V_{5(t)}=-\varphi-GR_tV_{2(t)}. \tag{13}$$

With the following boundary conditions:

$$\left.\begin{array}{l} V_{2(t)}=V_{2\alpha}, V_{5(t)}=0 \text{ on } S=1 \\ \dfrac{dV_{2(t)}}{dS_2}=\dfrac{dV_{5(t)}}{dS_5}=0 \end{array}\right\}. \tag{14}$$

Where GR_t is the growth-rate of assets, λ_1 *and* λ_2 represents rate of returns of first and second investments respectively, $\lambda_1+\lambda_2$ is additive effects and $\left(\lambda_1+\lambda_2\right)^{-1}$ additive inverse effects.

3.2 Method of Solution

The propose model (12) and (13) consist of a system of variable coefficient differential equations whose solutions are not trivial. Firstly we solve equation (12), (13) and substitute (12) into right hand side of (13) then solving to obtain particular solution. So solving (12) using Ferobenius method.

$$\text{Let } V_{2(t)}=\sum_{m=0}^{\infty}a_mS_2{}^{m+c}=S_2{}^c\sum_{m=0}^{\infty}a_mS_2{}^m, \tag{15}$$

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$$V'_{2(t)} = \sum_{m=0}^{\infty} a_m (m+c) S_2^{\,m+c-1} = S_2^{\,c=1} \sum_{m=0}^{\infty} a_m (m+c) S_2^{\,m}, \tag{16}$$

$$V''_{2(t)} = S_2^{\,c-2} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_2^{\,m} = S_2^{\,c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_2^{\,m}, \tag{17}$$

Substituting (15)-(17) into (12) gives:

$$S_2^{\,c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_2^{\,m} + S^{c-1} \sum_{m=0}^{\infty} a_m (m+c) S_2^{\,m} - \sum_{m=0}^{\infty} (\lambda_1 + \lambda_2)^2 \sigma^2 a_m S_2^{\,m+c+1} = 0, \tag{18}$$

$$S_2^{\,c-1} \sum_{m=0}^{\infty} a_m (m+c)^2 S_2^{\,m} - \sum_{m=0}^{\infty} (\lambda_1 + \lambda_2)^2 \sigma^2 a_m S_2^{\,m+c+1} = 0. \tag{19}$$

Setting $k = n-1 \Rightarrow n = k+1$ and carrying out some simplifications gives:

$$V_{2(t)}(S) = a_0 S^c \left\{ \begin{array}{l} 1 + \dfrac{(\lambda_1+\lambda_2)^2 \sigma^2 a_0 S_2^{\,2}}{(c+2)^2} + \dfrac{(\lambda_1+\lambda_2)^2 \sigma^4 a_0 S_2^{\,4}}{(c+2)^2 (c+4)^2} \\ + \dfrac{(\lambda_1+\lambda_2)^2 \sigma^6 a_0 S_2^{\,6}}{(c+2)^2 (c+4)^2 (c+6)^2} + \dfrac{(\lambda_1+\lambda_2)^2 \sigma^8 a_0 S_2^{\,8}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} \\ + \dfrac{(\lambda_1+\lambda_2)^2 \sigma^{10} a_0 S_2^{\,10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} \end{array} \right\}, \tag{20}$$

When $c = 0$ (20) becomes:

$$V_{2(t)}(S_2) = v_1 = \phi_1 \left\{ \begin{array}{l} 1 + \dfrac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2^{\,2}}{2^2} + \dfrac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2^{\,4}}{2^2 \times 4^2} + \dfrac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2^{\,6}}{2^2 \times 4^2 \times 6^2} \\ + \dfrac{(\lambda_1+\lambda_2)^8 \sigma^8 S_2^{\,8}}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dfrac{(\lambda_1+\lambda_2)^{10} \sigma^{10} S_2^{\,10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{array} \right\}, \tag{21}$$

Another is given by $v_2 = \dfrac{dV_{2(t)}}{dc}$ and also talking care of some simplifications gives:

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$$V_{2(t)} = v_2 = \phi_2 \left\{ \begin{array}{l} \ln S_2 \left(\begin{array}{l} 1+\frac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2{}^2}{2^2}+\frac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2{}^4}{2^2\times 4^2}+\frac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2{}^6}{2^2\times 4^2\times 6^2} \\ +\frac{(\lambda_1+\lambda_2)^8 \sigma^8 S_2{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\frac{(\lambda_1+\lambda_2)^{10} \sigma^{10} S_2{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2} \end{array} \right) \\ +a_0 S_2{}^c \left(\begin{array}{l} -\frac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2{}^2}{2^2}-\frac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2{}^4}{2^3\times 4^2} \\ -\frac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2{}^6}{4^3\times 6^3}+\frac{(\lambda_1+\lambda_2)^8 \sigma^8 S_2{}^8}{2^5\times 6^3\times 8^3} \\ -\frac{(\lambda_1+\lambda_2)^{10} \sigma^{10} S_2{}^{10}}{4^2\times 6^3\times 8^3\times 10^3}+\ldots \end{array} \right) \end{array} \right\}, \quad (22)$$

A linear combination (21) and (22) gives the complete solution:

$$V_{2(t)}(S_2) = \phi_1 \left\{ \begin{array}{l} 1+\frac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2{}^2}{2^2}+\frac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2{}^4}{2^2\times 4^2}+\frac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2{}^6}{2^2\times 4^2\times 6^2}+\frac{(\lambda_1+\lambda_2)^8 \sigma^8 S_2{}^8}{2^2\times 4^2\times 6^2\times 8^2} \\ +\frac{(\lambda_1+\lambda_2)^{10} \sigma^{10} S_2{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots \end{array} \right\}$$

$$+\phi_2 \left\{ \begin{array}{l} \ln S_2 \left(\begin{array}{l} 1+\frac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2{}^2}{2^2}+\frac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2{}^4}{2^2\times 4^2}+\frac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2{}^6}{2^2\times 4^2\times 6^2}+\frac{(\lambda_1+\lambda_2)^8 \sigma^8 S_2{}^8}{2^2\times 4^2\times 6^2\times 8^2} \\ +\frac{(\lambda_1+\lambda_2)^{10} \sigma^{10} S_2{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2} \end{array} \right) \\ -\frac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2{}^2}{2^2}-\frac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2{}^4}{2^3\times 4^2}-\frac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2{}^6}{4^3\times 6^3}+\frac{(\lambda_1+\lambda_2)^8 \sigma^8 S_2{}^8}{2^5\times 6^3\times 8^3} \\ -\frac{(\lambda_1+\lambda_2)^2 \sigma^{10} S_2{}^{10}}{4^2\times 6^3\times 8^3\times 10^3}+\ldots \end{array} \right\}. \quad (23)$$

Applying the boundary conditions (13) and setting $\phi_2 = 0$ gives:

$$\phi_1 = \frac{V_{2a}}{y^*{}_1(1)}.$$

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where

$$y^*_1(1) = \left\{ \begin{array}{l} 1 + \frac{(\lambda_1+\lambda_2)^2 \sigma^2}{2^2} + \frac{(\lambda_1+\lambda_2)^4 \sigma^4}{2^2 \times 4^2} + \frac{(\lambda_1+\lambda_2)^6 \sigma^6}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1+\lambda_2)^8 \sigma^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{(\lambda_1+\lambda_2)^{10} \sigma^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \dots \end{array} \right\}.$$

$$V_{2(t)}(S_2) = \frac{V_{2a}}{y^*_1(1)} \left\{ \begin{array}{l} 1 + \frac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2{}^2}{2^2} + \frac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2{}^4}{2^2 \times 4^2} + \frac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2{}^6}{2^2 \times 4^2 \times 6^2} \\ + \frac{(\lambda_1+\lambda_2)^6 \sigma^8 S_2{}^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{(\lambda_1+\lambda_2)^{10} \sigma^{10} S_2{}^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \dots \end{array} \right\}. \quad (24)$$

Solving the homogenous part of (13) by assuming a solution:

Let $V_{5(t)} = \sum_{m=0}^{\infty} a_m S_5{}^{m+c} = S_5{}^c \sum_{m=0}^{\infty} a_m S_5{}^m.$ (25)

$$V'_{5(t)} = \sum_{m=0}^{\infty} a_m (m+c) S_5{}^{m+c-1} = S_5{}^{c=1} \sum_{n=0}^{\infty} a_m (m+c) S_5{}^m, \quad (26)$$

$$V''_{5(t)} = S_5{}^{c-2} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_5{}^m = S_5{}^{c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_5{}^m, \quad (27)$$

substituting (25)-(27) into (13) gives:

$$S_5{}^{c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_5{}^m + S_5{}^{c-1} \sum_{m=0}^{\infty} a_m (m+c) S_5{}^m - \sum_{m=0}^{\infty} \left\{ (\lambda_1+\lambda_2)^{-1} \right\}^2 \sigma^2 a_m S_5{}^{m+c+1} = 0, \quad (28)$$

$$S_5{}^{c-1} \sum_{m=0}^{\infty} a_m (m+c)^2 S_5{}^m - \sum_{m=0}^{\infty} \left\{ (\lambda_1+\lambda_2)^{-1} \right\}^2 \sigma^2 a_m S_5{}^{m+c+1} = 0. \quad (29)$$

Setting $k = n-1 \Rightarrow n = k+1$ and carrying out some simplifications gives:

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$$V_{5(t)}(S_5) = a_0 {S_5}^c \begin{Bmatrix} 1 + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 a_0 {S_5}^2}{(c+2)^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 a_0 {S_5}^4}{(c+2)^2 (c+4)^2} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 a_0 {S_5}^6}{(c+2)^2 (c+4)^2 (c+6)^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 a_0 {S_5}^8}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} a_0 {S_5}^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} \end{Bmatrix}. \qquad (30)$$

When $c = 0$ (30) becomes:

$$V_{5(t)}(S_5) = v_1 = \alpha_1 \begin{Bmatrix} 1 + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 {S_5}^2}{2^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 {S_5}^4}{2^2 \times 4^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 {S_5}^6}{2^2 \times 4^2 \times 6^2} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 {S_5}^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} {S_5}^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \dots \end{Bmatrix}.$$

(31)

Another is given by $v_2 = \dfrac{dV_{1(t)}}{dc}$ and talking long simplifications gives.

$$V_{5(t)} = v_2 = \alpha_2 \begin{Bmatrix} \ln S_5 \begin{pmatrix} 1 + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 {S_5}^2}{2^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 {S_5}^4}{2^2 \times 4^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 {S_5}^6}{2^2 \times 4^2 \times 6^2} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 {S_5}^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} {S_5}^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{pmatrix} \\ + a_0 {S_5}^c \begin{pmatrix} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 {S_5}^2}{2^2} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 {S_5}^4}{2^3 \times 4^2} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 {S_5}^6}{4^3 \times 6^3} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 {S_5}^8}{2^5 \times 6^3 \times 8^3} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} {S_5}^{10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \dots \end{pmatrix} \end{Bmatrix}.$$

(32)

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A linear combination (31) and (32) gives the complete solution:

$$V_{5(t)}(S_5)=\alpha_1\left\{\begin{array}{l}1+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2\sigma^2 S_5{}^2}{2^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4\sigma^4 S_5{}^4}{2^2\times 4^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6\sigma^6 S_5{}^6}{2^2\times 4^2\times 6^2}\\ +\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8\sigma^8 S_5{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10}\sigma^{10} S_5{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots\end{array}\right\},$$

$$+\alpha_2\left\{\begin{array}{l}\ln S_5\left(\begin{array}{l}1+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2\sigma^2 S_5{}^2}{2^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4\sigma^4 S_5{}^4}{2^2\times 4^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6\sigma^6 S_5{}^6}{2^2\times 4^2\times 6^2}\\ +\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8\sigma^8 S_5{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10}\sigma^{10} S_5{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}\end{array}\right)\\ -\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2\sigma^2 S_5{}^2}{2^2}-\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4\sigma^4 S_5{}^4}{2^3\times 4^2}-\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6\sigma^6 S_5{}^6}{4^3\times 6^3}\\ +\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8\sigma^8 S_5{}^8}{2^5\times 6^3\times 8^3}-\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10}\sigma^{10} S_5{}^{10}}{4^2\times 6^3\times 8^3\times 10^3}+\ldots\end{array}\right\} \qquad (33)$$

Applying the boundary conditions (14) and setting $\alpha_2=0$ gives

$$V_{5(t)}(S_5)=\left\{\begin{array}{l}1+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2\sigma^2 S_5{}^2}{2^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4\sigma^4 S_5{}^4}{2^2\times 4^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6\sigma^6 S_5{}^6}{2^2\times 4^2\times 6^2}\\ +\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8\sigma^8 S_5{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10}\sigma^{10} S_5^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots\end{array}\right\}. \qquad (34)$$

From Right hand side (RHS) of (13) using the method of undetermined coefficient.

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$$V_{4(t)}(S_4)=\alpha_1\begin{Bmatrix}1+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^2\sigma^2 S_4{}^2}{2^2}+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^4\sigma^4 S_4{}^4}{2^2\times 4^2}+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6\sigma^6 S_4{}^6}{2^2\times 4^2\times 6^2}\\+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6\sigma^8 S_4{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^{10}\sigma^{10} S_4{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots\end{Bmatrix}.$$

The complementary function for (13) is given as follows:

$$V_{5(t)c}(S_5)=C_c\begin{bmatrix}1+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_5)\right\}^2}{2^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_5)\right\}^4}{(2\times 4)^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_5)\right\}^6}{(2\times 4\times 6)^2}\\+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_5)\right\}^8}{(2\times 4\times 6\times 8)^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_5)\right\}^{10}}{(2\times 4\times 6\times 8\times 10)^2}+\ldots\end{bmatrix}, \quad (35)$$

Consider the particular solution

$$V_p(S)=A_0+A_1S^2+A_2S^4+A_3S^6+A_4S^8+A_5S^{10}, \quad (36)$$

$$V_p'(S)=2A_1S+4A_2S^3+6A_3S^5+8A_4S^7+10A_5S^9, \quad (37)$$

$$V_p''(S)=2A_1+12A_2S^2+30A_3S^4+56A_4S^6+90A_5S^8, \quad (38)$$

Putting (36)-(38) into (13) gives the following:

$$\Rightarrow 4A_1+16A_2S^2+36A_3S^4+64A_4S^6+100A_5S^8-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_0-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_1S^2$$
$$-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_2S^4-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_3S^6-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_4S^8-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_5S^{10}, \quad (39)$$

$$-\varphi-GR_tV_{2(2)}\begin{pmatrix}1+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S\right\}^2}{2^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S\right\}^4}{(2\times 4)^2}+\dfrac{\left\{(\lambda+_1\lambda_2)^{-1}\sigma S\right\}^6}{(2\times 4\times 6)^2}\\+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S\right\}^8}{(2\times 4\times 6\times 8)^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S\right\}^{10}}{(2\times 4\times 6\times 8\times 10)^2}\end{pmatrix}, \quad (40)$$

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Combining (39) and (40) yields the following results:

$$4A_1 - \left\{\left(\lambda_1 + \lambda_2\right)^{-1}\right\} A_0 = -\varphi - GR_t V_{2(t)}, \tag{41}$$

$$16A_2 - \left\{(\lambda_1 + \lambda_2)^{-1}\sigma\right\}^2 A_1 = -\frac{\left\{(\lambda_1 + \lambda_2)^{-1}\sigma\right\}^2 GR_t V_{2(t)}}{4}, \tag{42}$$

$$36A_3 - \left\{\left(\lambda_1 + \lambda_2\right)^{-1} \sigma\right\}^2 A_2 = \frac{\left\{(\lambda_1 + \lambda_2)^{-1}\sigma\right\}^4 GR_t V_{2(t)}}{\left(2\times 4\right)^2}, \tag{43}$$

$$64A_4 - \left\{\left(\lambda_1 + \lambda_2\right)^{-1} \sigma\right\}^2 A_3 = \frac{\left\{(\lambda_1 + \lambda_2)^{-1}\sigma\right\}^6 GR_t V_{2(t)}}{\left(2\times 4\times 6\right)^2}, \tag{44}$$

$$100A_5 - \left\{\left(\lambda_1 + \lambda_2\right)^{-1} \sigma\right\}^2 A_4 = -\frac{\left\{(\lambda_1 + \lambda_2)^{-1}\sigma\right\}^8 GR_t V_{2(t)}}{\left(2\times 4\times 6\times 8\right)^2}, \tag{45}$$

$$-\left\{\left(\lambda_1 + \lambda_2\right)^{-1} \sigma\right\}^2 A_5 = -\frac{\left\{(\lambda_1 + \lambda_2)^{-1}\sigma\right\}^{10} GR_t V_{2(t)}}{\left(2\times 4\times 6\times 8\times 10\right)^2}. \tag{46}$$

where A_0, A_1, A_3, A_4 and A_5 are constant which can be seen in the appendix.

To get the constant c , we combine (35) and (36) to gives:

$$V_4\left(S\right) = c\left[\begin{array}{l} 1 + \frac{((\lambda_1 + \lambda_2)^{-1}\sigma)^2 S^2}{4} + \frac{((\lambda_1 + \lambda_2)^{-1}\sigma)^4 S^4}{\left(2\times 4\right)^2} + \frac{((\lambda_1 + \lambda_2)^{-1}\sigma)^6 S^6}{\left(2\times 4\times 6\right)^2} + \frac{((\lambda_1 + \lambda_2)^{-1}\sigma)^8 S^8}{\left(2\times 4\times 6\times 8\right)^2} \\ + \frac{((\lambda_1 + \lambda_2)^{-1}\sigma)^{10} S^{10}}{\left(2\times 4\times 6\times 8\times 10\right)^2} \end{array}\right]. \\ + A_0 + A_1 S^2 + A_2 S^4 + A_3 S^6 + A_4 S^8 + A_4 S^{10} \tag{47}$$

Applying boundary condition (14) on (47), doing some simplifications and talking like terms gives a complete solution.

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$$V_5(S_5) = c + A_0 + \left(\frac{c((\lambda_1+\lambda_2)^{-1}\sigma)^2}{4} + A_1\right)S_5^{\,2} + \left(\frac{c((\lambda_1+\lambda_2)^{-1}\sigma)^4}{(2\times 4)^2} + A_2\right)S_5^{\,4}$$

$$+\left(\frac{c((\lambda_1+\lambda_2)^{-1}\sigma)^6}{(2\times 4\times 6)^2} + A_3\right)S_5^{\,6} + \left(\frac{c((\lambda_1+\lambda_2)^{-1}\sigma)^8}{(2\times 4\times 6\times 8)^2} + A_4\right)S_5^{\,8} + \left(\frac{c((\lambda_1+\lambda_2)^{-1}\sigma)^{10}}{(2\times 4\times 6\times 8\times 10)^2} + A_5\right)S_5^{\,10}. \tag{48}$$

4. RESULTS AND DISCUSSION

This Section presents the graphical results for whose solutions are in (24) and (48) respectively. Hence the following parameter values were used in the simulation study:

$\lambda_1 = 180,\ \lambda_2 = 120\ , \sigma = 0.3, \mathrm{V}_{2\alpha} = 0.2$ and $c = 1.0$.

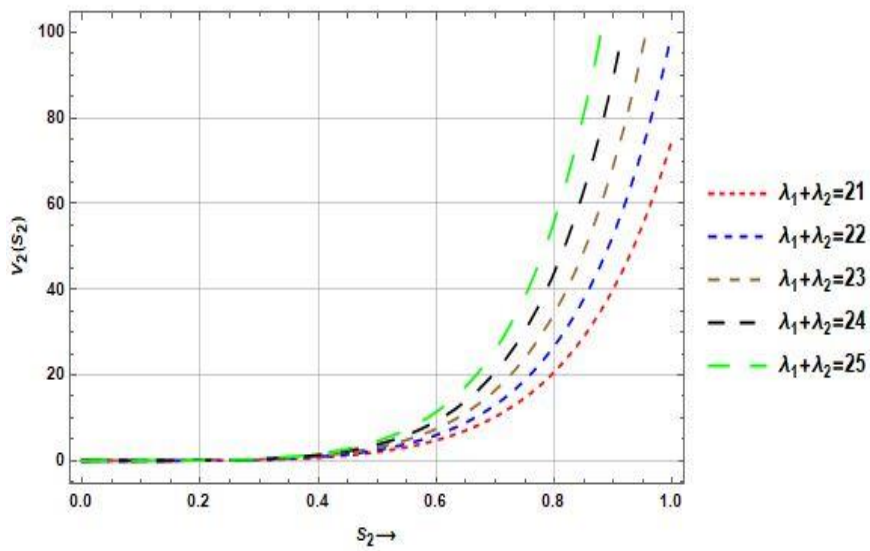

Figure 1: The plot of asset value when return rate follows additive effects

Figure 1 show the variation of return rate parameter when it follows additive effects. It can be observed that increasing return rates when stock return rates follows additive effects also increases the value of asset pricing throughout the specified trading periods. This suggests that additive effect is profit maximizing in terms of long and short term investment plans. It further informs investors the positive pictures of the future investments hence the return rate is additive.

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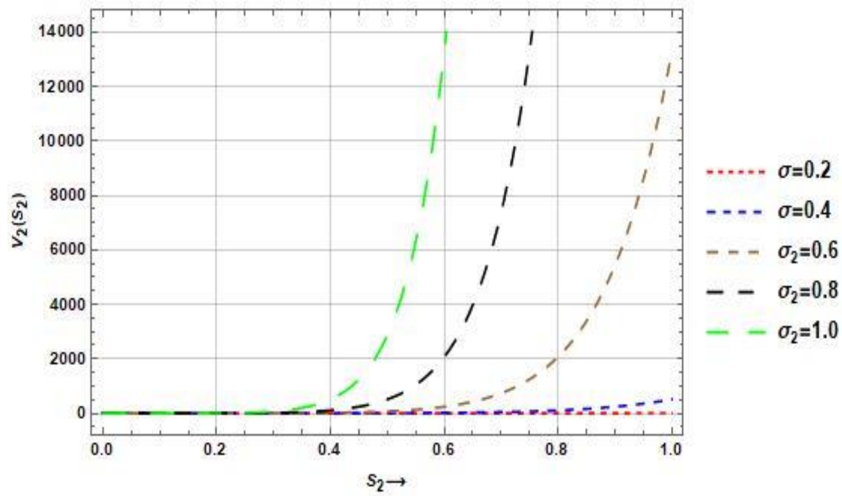

Figure 2: The plot of asset value with variations of volatility parameter

Figure 2 show plot of asset value with different forms of stock volatility parameter. It can be noticed that increase in stock volatility increases the levels of return rates of an investment over several periods of time. This is correct because as cost of commodities in the markets are increasing, that is how various levels of investments will also increase; hence volatility causes significant changes on the value of asset over time. In another dimension the plots shows an exponential curve which displays stochastic formation in the price history of stock market. This type of investments is profit maximizing because is under-going positive increase on day to day activities of the trading business. The same interpretation holds for Figure 7.

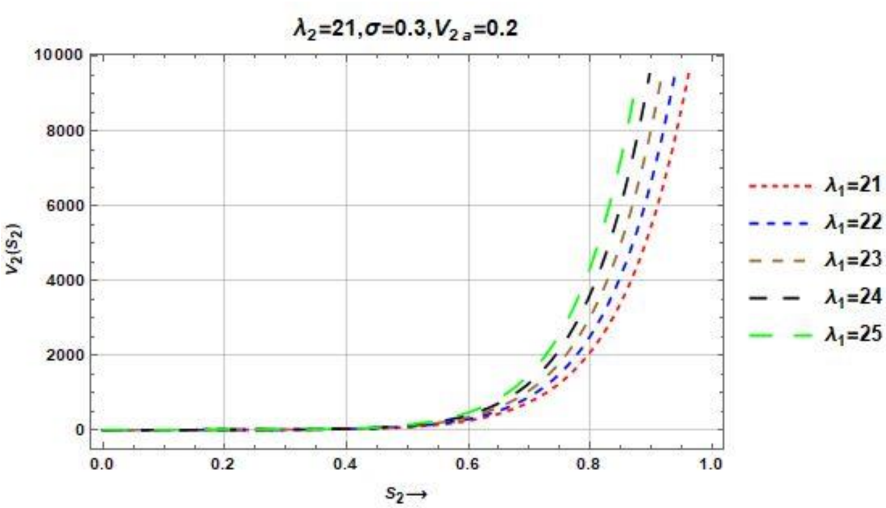

Figure 3: The plot of asset value when first return rate of the investment follows additive effects

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Figure 3 and 4 show increase in return rates increases the value of asset which is of good benefit to investors because it measures the profit of the investment over time. The implication of these results is that it indicates the effectiveness of the financial allocation of the investments and accomplishing social, environmental, economic or governance related objectives across every unit within the organization.

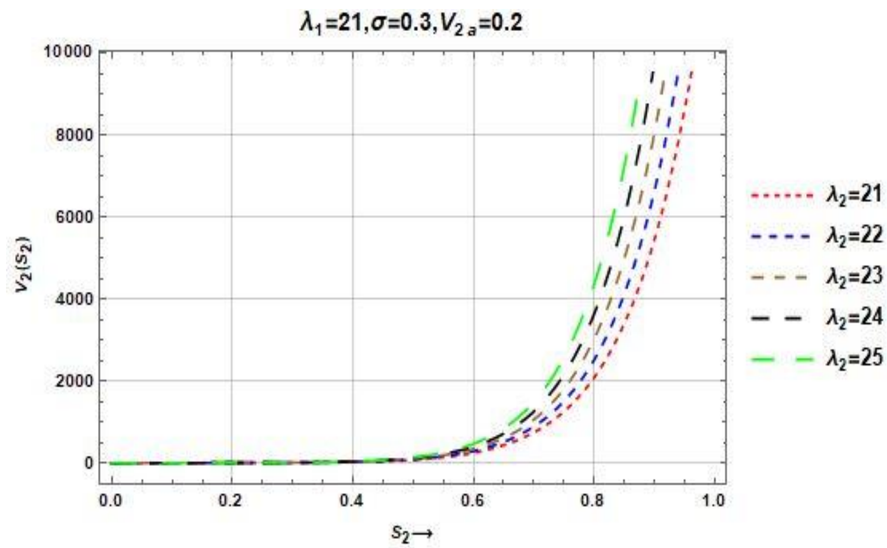

Figure 4: The plot of asset value when second return rate of the investment follows additive effects

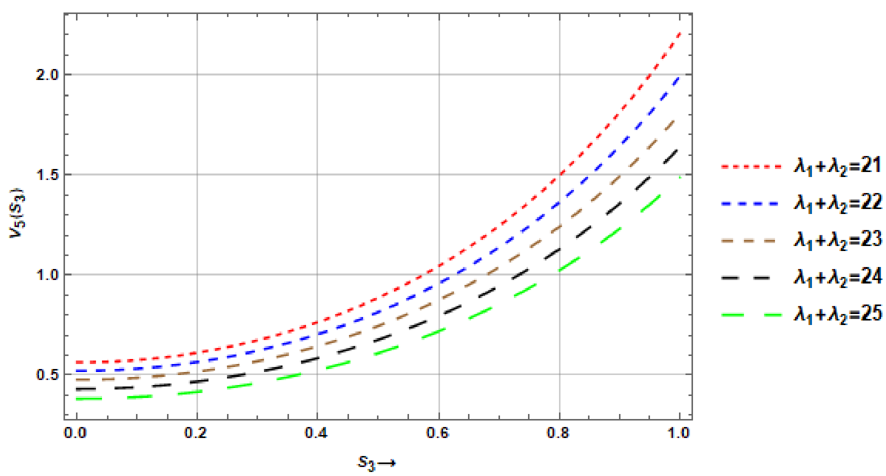

Figure 5: The plot of asset value when return rate follows additive inverse effects

In Figure 5, it is observed that increase in additive inverse rate of return decreases the value of asset over time. This situation can only take place where levels of return rate is not capable enough to sustain the strength of the entire workers; as result of either excess debts owed by the particular investment or over employment of workers , lack of financial appropriation by the management of the investment.

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All of these cause great decrease in return rates. This informs investors to make viable decisions based on the levels of their investments. Finally, the natural nature of inverse as a term is play major roles as it affects financial variables.

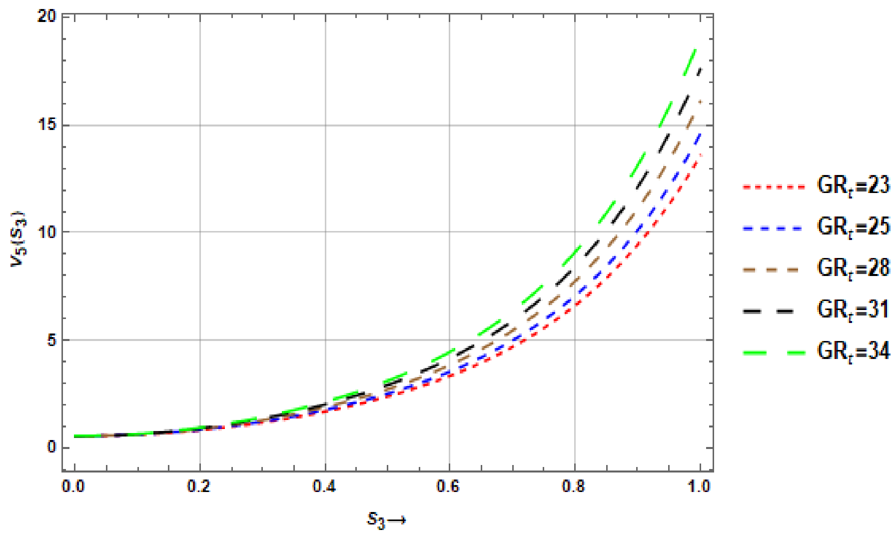

Figure 6: The plot of asset value with variations of Growth-rate parameters

Figure 6 show the plot of asset value with different growth rate parameters. It can be seen that a little increase in the growth rate parameter increases the value of asset of capital investment. This is quite encouraging because it assesses the investment performance and predicting the future profit of an investment over time. This remark is crucial for a long-term business plans because it attracts enough funds in the investments for human capital development.

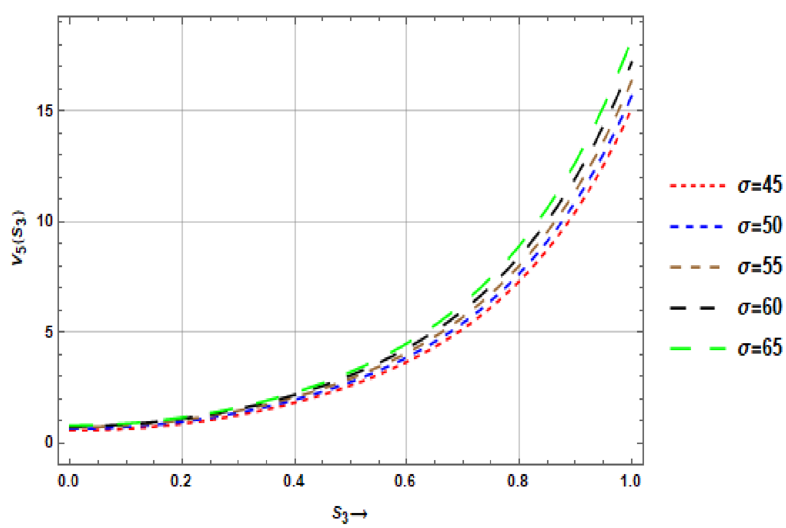

Figure 7: The plot of asset value with variations volatility parameter

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5. CONCLUSION AND RECOMMENDATIONS

The success of any investment hinges mainly on the value of asset which urges the whole financial strength of all investments and differential equations are well recognized predominant mathematical instrument used for the prediction of stock market variables. Therefore, we considered system of coupled differential equations with some stochastic financial market variables in the model. These problems were solved analytical by adopting the Ferobenius method of series solution and two different investment solutions were obtained accurately. The necessary conditions were achieved which govern asset price return rates through additive effects, additive inverse effects, and asset growth-rates of assets parameter respectively. From the analysis we deduce that (i) additive rate of returns is better used as a measure in respect to asset value than additive inverse effects (ii) increase in the growth rate parameter increases the value of asset of capital investment (iii) exponential growth of financial assets are profit maximizing in time varying investments.

Consequently, we recommend that combining ordinary differential and stochastic differential equations in the assessment of asset value and its return rates will be an interesting study to explore.

APPENDIX 1. CONSTANTS OF THE INVESTMENT

$$\Rightarrow A_5 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[\frac{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^{10} GR_t V_{2(t)}}{\left(2\times4\times6\times8\times10\right)^2}\right],$$

$$\Rightarrow A_4 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[100A_5 + \frac{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^{8} GR_t V_{2(t)}}{\left(2\times4\times6\times8\right)^2}\right]$$

$$\Rightarrow A_3 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[64A_4 - \frac{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^{6} GR_t V_{2(t)}}{\left(2\times4\times6\right)^2}\right],$$

$$\Rightarrow A_2 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[36A_3 - \frac{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^{4} GR_t V_{2(t)}}{\left(2\times4\right)^2}\right]$$

$$\Rightarrow A_1 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[16A_2 - \frac{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^{2} GR_t V_{2(t)}}{4}\right],$$

EQUATION $$\Rightarrow A_0 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[4A_1 + \varphi - GR_t V_{2(t)}\right]$$

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