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A Theoretical Cauchy-Bunyakovsky-Schwartz Inequality Estimation of Upper Bound for the Expected Claim per Loss Payment in Non-Life Insurance Business

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ABSTRACT

In an under-explored insurance market, it is necessary to estimate claim severity under deductible conditions. The aim of this paper is to theoretically construct an upper bound for the mean severity through an alternative technique of Cauchy-Bunyakovsky-Schwartz inequality which can play a central role in grey areas of applied casualty analysis. We identified, interrelate and applied discrete and continuous inequalities which are connected with Cauchy's inequality and provide novel framework to study functional characteristics associated with actuarial severities. The result show that the expected claim per loss payment in general insurance business under potential kernel with deductible clauses D can be estimated by

$$\langle Y_L \rangle \leq 0.5\left[\left(\langle y^2 \rangle - 2D\langle y \rangle + D^2\right)^{0.5} + \left(\langle (Y-D)_+ \rangle - \langle (D-Y)_+ \rangle\right)\right]$$

Keywords: deductible, dirac-delta function, indemnity, singularity

1. INTRODUCTION

Underwriting business is generally sensitive to macro-economic variables and often considers economic fundamentals in order to choose the correct investment vehicles. These fundamentals are inflationary index, interest rates trajectories, foreign exchange reserves, government debt and exchange rates that continue unabated as a result of hostile fiscal policies. Insurance market in under-developed economies such as Nigeria is usually unexplored as result of huge outstanding claims, inadequate knowledge of insurance principles by the insuring public, lack of skilled underwriters, weak regulatory environment, income inequalities and inaccessibility to information technology**.** Furthermore, underwriters are not encouraged to invest insurance premiums in long-term investible instruments due to hyper-inflationary trends experienced up till date as a result of hostile fiscal environment. If this trend remains unabated, underwriters cannot solve liquidity-related issues which discourage insurers to honor outstanding claims. It is therefore necessary to estimate claim severity under deductible modifications. In order to solve and justify this problem, we introduce singularity potential and Cauchy-Bunyakovsky-Schwartz inequality as our working tools in the estimation of upper bound for the expected claim per loss payment in non-life insurance business. The present study will be markedly significant on the following grounds where: Underwriters explore to invest large resources in developing actuarial models in non-life with much interest on the correct pricing of insured risk including risk profiles which seems quite complex to quantify and which are characterized by extra risk features but which could much more influence the insurance market drivers. In computing technical provisions, the technique developed here in estimating the cost per loss shall be sufficiently applicable and relevant under credible actuarial assumptions so as to enable underwriters justify the actuarial basis to the regulatory bodies. The model described will be sufficient to cover non-life and health insurance risk profile especially where the underwriter searches for the optimal solution strategy to the common problems of premium pricing, products development, risk rating and quantification, capital adequacy based on solvency requirements, valuation of insurance funds and management of insurance capital. In an insurance business environment where the future is uncertain with potential for much impact possibly, the behavior of the insured will then seem to depend on expectations and consequently, the actuary would need to express the uncertainty of the actual insurance environment in terms of random risk using mathematical expectation and singularity potential as working tools so as to obtain a fairly acceptable underwriting pattern. Real valued function $U(x)$ which defines a value at all points x and representing physical phenomena may not really translate to a measurable function in practical applications. For instance in measuring instantaneous acceleration of a particle at a point, given accurate and vast measuring tools could be functionally difficult unless the uniform average values of $U(x)$ in a small interval of time is taken which is equivalent to the integral $\int_{-\varepsilon}^{\varepsilon} U(x)\,dx$. Because, uniformly averaging of $U(x)$ over $(-\varepsilon,\varepsilon)$ may not be feasible in the domain of definition, test function $\zeta(x)$ called weightings could then be assigned since the measuring tools may be somewhat sensitive at some definite points

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and consequently the average value becomes $\int_{-\varepsilon}^{\varepsilon} \zeta(x)U(x)dx$ leading to regular distribution function where $\zeta : R \to R$ is infinitely differentiable and non-zero in a finite region. Because not every real valued function defines regular distribution, it is required that $\int_{-\infty}^{\infty} \zeta U$ be finite for all ζ in a region O.

2. LITERATURE REVIEW

The dirac-delta function as inferred from (Mohammed, 2011; Brewster & Franson, 2018; Ogungbenle et al, 2020) is a distribution function which provides a convenient way of describing the singularity structure of certain probability distributions. Because it is used to get insight into characteristics of a system in a mixed condition, it could be applied to compute the properties of an actuarial risk process which would have been complex to obtain using other methodologies. We see in (Khuri, 2004; Onural, 2006; Chakraborty, 2008; Mohammed, 2011; Ogungbenle et al, 2020), that $\delta(x)$ is singular at a point x on the real line where the operator integral $\int_R dx\delta(x)dx$ evaluates the functional value of a function at zero. In (Chakraborty, 2008; Sastry, 2009; Mohammed, 2011; Salasnich, 2014; Zhang, 2018), we observe that $\delta(x)$ describes the most convenient right-hand side in solving differential equations and consequently this may lead to Green functions of the form

$G(s,\zeta) = H(s-\zeta)e^{-(s-\zeta)}$ where $H(s-\zeta)$ is the heavy side. Dirac-delta function is a generalized function such that for a nice function $g(.)$, its derivatives

$$\frac{d}{dx}g(x), \frac{d^2}{dx^2}g(x), \ldots \text{ all tends to zero as } x \to \infty \tag{1}$$

Consider the function
$$\eta(y,k) = \begin{cases} a & if \quad k - \frac{1}{2a} < y < k + \frac{1}{2a} \\ 0 & if \quad otherwise \end{cases} \tag{2}$$

The integral
$$\int_c^d \eta(y,k)dy \to 1 \tag{3}$$

if
$$k - \frac{1}{2a} < y < k + \frac{1}{2a} \subset (c,d) \tag{4}$$

but it will be zero if otherwise.

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$$\delta(y-k)=\lim_{a\to\infty}\eta(y,k)=\lim_{a\to\infty}\eta(y-k) \quad (5)$$

If $f(x)$ defines any continuous function over the interval (c,d) then by the mean value theorem for integrals, we have

$$\int_c^d \eta(y,k)f(y)dy=\int_{k-\frac{1}{2a}}^{k+\frac{1}{2a}}\eta(y,k)f(y)dy=\left(\left(k+\frac{1}{2a}\right)-\left(k-\frac{1}{2a}\right)\right)f(\varsigma)h(\varsigma) \quad (6)$$

$$=\frac{1}{a}f(\varsigma)\times a=f(\varsigma)$$

where $\varsigma\in\left(k-\frac{1}{2a}<y<k+\frac{1}{2a}\right)$ and $h(\varsigma)=a$ in the interval

As $a\to\infty$, then $\eta(y,k)\to\delta(y-k)$ (7)

and at the same time the point ς in $\left(k-\frac{1}{2a}<y<k+\frac{1}{2a}\right)$ will tend to k and consequently,

$$f(\varsigma)=f(k) \quad (8)$$

From the arguments above and following (Pazman & Pronzato, 1996; Kanwal, 1998; Salasnich, 2014), the dirac-delta function is then defined by

$$\int_c^d f(y)\delta(y-k)dy=\begin{cases} f(k) & if \quad c<k<d \\ 0 & if \quad otherwise\end{cases} \quad (9)$$

3. MATERIAL AND METHODS

We follow methodology in (Tse, 2009; Ogungbenle et al, 2020). Consider

$$x_a<t_0<x_b, \int_{x_a}^{x_b}\delta(x-t_0)g(x)dx=\int_{x_a}^{x_b}\delta(x-t_0)g(t_0)dx=g(t_0)\int_{x_a}^{x_b}\delta(x-t_0)dx \quad (10)$$

$$\int_{x_a}^{x_b}\delta(x-t_0)g(x)dx=g(t_0) \quad (11)$$

If $t_0=0$, we have $\int_{x_a}^{x_b}\delta(x)g(x)dx=g(0)$ (12)

Define $H(x-a)=h_a(x)$ (12a)

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If $H(x)$ is the unit step function, then $\delta(x-a)dx = dh_a(x)$ (13)

Equation (12a) can be modified as $\varsigma\delta_{\varsigma}(x-x_a) = H(x-x_a) - H(x-x_a-\varsigma)$ (13a)

$$\Rightarrow \varsigma\delta_{\varsigma}(x-x_a) = h_{x_a}(x) - h_{x_a+\varsigma}(x)$$

When $x_a = 0$ and taking limit, we have $\lim_{\varsigma\to 0}\delta_{\varsigma}(x) = \lim_{\varsigma\to 0}\frac{h_0(x)-h_{\varsigma}(x)}{\varsigma} = \frac{dH}{dx}$

If an arbitrary function $f(x)$ exists in $[x_a, x_b]$ but turns zero outside the interval, then it could be
expressed as $\left[h_{x_a}(x) - h_{x_b}(x)\right] f(x) = w(x)$ where $w(x)$ is continuous

Let $G_X(x)$ be the distribution function of a random risk X with the property that
$dG_X(x) = g_X(x)dx$ (14)

Define $G_X(x) = \sum_{x_i\in\Omega_X} P(x_i)h_{x_i}(x)$, where Ω_X is the support of X. By the above property,

$$\frac{dG_X(x)}{dx} = \frac{d}{dx}\left[\sum_{x_i\in\Omega_X} P(x_i)h_{x_i}(x)\right] = \left[\sum_{x_i\in\Omega_X} P(x_i)\frac{d}{dx}\left[h_{x_i}(x)\right]\right], \tag{15}$$

so that the probability density function is obtained as

$$\frac{dG_X(x)}{dx} = g_X(x) = \sum_{x_i\in\Omega_X} P(x_i)\delta(x-x_i) \tag{16}$$

where $\Omega_X = \{x_i\}\ i = 1, 2, 3, \ldots$ and $P(x_i)$ are the probability mass points. Let us now compute the moments of the random risk X.

$$E(X) = \int_{-\infty}^{\infty} sf_X(s)d = \int_{s=0}^{\infty}\left[\int_{y=0}^{s} dy\right]dF_X(s) = \int_{x=0}^{\infty}\int_{t=x}^{\infty} dF_X(s)dx = \int_{0}^{\infty} S_X(s)dx \tag{16a}$$

$$E(X) = \int_{-\infty}^{\infty} xg_X(x)dx = \int_{-\infty}^{\infty}\left[x\sum_{x_i\in\Omega_X} P(x_i)\delta(x-x_i)\right]dx \tag{17}$$

$$E(X) = \int_{-\infty}^{\infty}\left[x\sum_{x_i\in\Omega_X} P(x_i)\delta(x-x_i)\right]dx = \left[\sum_{x_i\in\Omega_X} P(x_i)\int_{-\infty}^{\infty} x\delta(x-x_i)dx\right] \tag{18}$$

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$$E(X)=\left[\sum_{x_i\in\Omega_X} x_i P(x_i)\int_{-\infty}^{\infty}\delta(x-x_i)dx\right]=\left[\sum_{x_i\in\Omega_X} x_i P(x_i)\right] \tag{19}$$

$$E(X^m)=\int_{-\infty}^{\infty} x^m g_X(x)dx=\int_{-\infty}^{\infty}\left[\sum_{x_i\in\Omega_X} P(x_i)\delta(x-x_i)\right]x^m dx \tag{20}$$

$$E(X^m)=\left[\sum_{x_i\in\Omega_X} P(x_i)\int_{-\infty}^{\infty} x^m\delta(x-x_i)dx\right]=\sum_{x_i\in\Omega_X} x_i^m P(x_i) \tag{21}$$

Clearly, $$Var(X)=E(X^2)-(E(X))^2=\left[\sum_{x_i\in\Omega_X} x_i^2 P(x_i)\right]-\left[\sum_{x_i\in\Omega_X} x_i P(x_i)\right]^2 \tag{22}$$

3.1 The Insurance Risk Measures

Insurance risks are defined by random variables describing indemnity payable by the underwriting firm as a result of the occurrence of an insured peril. We let Y represent a random insurance variable describing the magnitude of a loss incurred by a risk. This is the random sum of monetary units the underwriter will pay the scheme holder over the execution of the insurance scheme. If Y is the total amount of claim incurred by the scheme holder, then following (Tse, 2009), the distribution of Y is as follows

$$G_Y(y)=\Pr(Y\le y)=\Pr\left\{Y^{-1}\left((-\infty,y]\right)\right\}, y\in R$$

$$G_Y(x)\le G_Y(y), x\le y, and, G_Y(\infty)=1$$

$$G_Y(y)=\lim_{\delta y\to 0+} G_Y(y+\delta y), and, G_Y(y)=\lim_{\delta y\to 0+} G_Y(y-\delta y)=\Pr(Y<y)$$

G is the probability that the scheme holder incurs a total claim of at most y and $g_Y(y)$ is the density. If Y is the risk covered by the insurance firm in exchange of premium P, then the loss relating to the risk is $L=Y-P$ where P is the discounted value of premium fixed by the terms of the policy so that the random consequences of insured peril are substituted with premium P. The random function $y\delta G_Y(y)$ is the density of an area under the graph of y plotted against $G_Y(y)$ and describes the contribution from losses within the interval $(y, y+\delta y)$. Suppose all such areas are added together, we obtain the expected loss incurred by the risk $\langle y\rangle=\int_{-\infty}^{+\infty} y dG_Y(y)$ enveloping

Severities of different magnitudes. In insurance practice, the excess of loss insurance covers massive combination of excess employer's liability, excess product liability, and excess public liability and with minimum premium, it is cost effective for potential insuring public of diverse background. It is sufficient that accuracy, quick response to clients and convenience is required in quoting excess liability cover for the insured and consequently, this is the reason why rapid quotations can be generated momentarily so that it may not be necessary to refer to primary wordings before going on coverage. With maximum amount of liability which an insurer assumes from insurance operations underwriting capacity, a high proportion of clients' excess liability exposures can be underwritten relaxing the need to impose any complicated co-insurance and multiple excess layer policies. Furthermore, highly professional claims managers pay attention wholly on treating clients' large and complex cases. Excess of loss solution provides valuable additional riders including medical services. Given an insured risk Y and any positive real deductible D, the expected part of loss Y greater than D called the excess of loss is defined as

$$\Sigma_Y(D) = \int_{-\infty}^{\infty} Max(y-D;0)dG_Y(y)$$

3.2 Theorem

If $\Sigma_Y(D) = \int_{-\infty}^{\infty} Max(y-D;0)\big(G_Y(y)-G_Y(y-\delta y)\big)$

where $\delta y > 0$ is an infinitesimal number and

$$\Sigma_Y(D) = \int_{y=D}^{\infty} S_Y(y)dy, and, \Sigma_Y(D+\sigma) = \Sigma_{Y-\sigma}(D), \sigma \in R \tag{23}$$

Proof

$G_Y(y)-G_Y(y-\delta y) \to dG_Y(y)$ represents the probability of y and S is the survival function

$$\Sigma_Y(D) = \int_{y=D}^{\infty} (y-D)dG_Y(y) \tag{24}$$

$$\Sigma_Y(0) = \int_{y=0}^{\infty} (y-0)dG_Y(y) = \int_{y=0}^{\infty} yg_Y(y)dy = E(y) \tag{25}$$

$$\Sigma_Y(D) = \int_{y=D}^{\infty} (y-D)dG_Y(y) = -\int_{y=D}^{\infty} (y-D)dS_Y(y) \tag{26}$$

$$\Sigma_Y(D) = -(y-D)S_Y(y)\Big|_D^{\infty} + \int_{y=D}^{\infty} S_Y(y)d(y-D) \tag{27}$$

$$\Sigma_Y(D) = -(y-D)S_Y(y)\Big|_D^{\infty} + \int_{y=D}^{\infty} S_Y(y)dy \tag{28}$$

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$$\Sigma_Y(D)=\int_{y=D}^{\infty} S_Y(y)dy=\int_{y=0}^{\infty} S_Y(y)dy-\int_{y=0}^{D} S_Y(y)dy \tag{29}$$

By definition,

$$\Sigma_{Y-\sigma}(D)=\int_{-\infty}^{\infty} Max(y-\sigma-D;0)dG_Y(y) \tag{30}$$

Because of the deductible condition, we integrate over the interval $\left[D+\sigma,\infty\right)$ (31)
using equation (26)

$$\Sigma_{Y-\sigma}(D)=\int_{y=(D+\sigma)}^{\infty} \left(y-(D+\sigma)\right)dG_Y(y) \tag{32}$$

$$\Sigma_Y(D+\sigma)=\int_{-\infty}^{\infty} Max\left(y-(D+\sigma);0\right)dG_Y(y) \tag{33}$$

$$\Sigma_Y(D+\sigma)=\int_{y=(D+\sigma)}^{\infty} \left(y-(D+\sigma)\right)dG_Y(y) \tag{34}$$

Therefore,

$$\Sigma_Y(D+\sigma)=\Sigma_{Y-\sigma}(D) \tag{35}$$

hence the proof

$$D=-\sigma\Rightarrow\int_{y=0}^{\infty}\left(y-(0)\right)dG_Y(y)=\mu_Y \tag{36}$$

3.3 Consequences of Definition of Excess of Loss

As a result of the above definition of excess of loss and equation (24), suppose $\left\langle(y)\right\rangle$ is the risk mean loss and μ_D the mean deductible, then by definition

$$\int_{-\infty}^{\mu_D}\left\langle(y-D)_+\right\rangle dy=\int_{-\infty}^{\mu_D}\int_{-\infty}^{y} G_D(D)dDdy \tag{37}$$

Again, by definition,

$$2\int_{-\infty}^{\mu_D}\left\langle(y-D)_+\right\rangle dy=\int_{-\infty}^{\mu_D}(D-\mu_D)^2\, g_D(D)dD;G'_D=g_D \tag{38}$$

Also $2\int_{\mu_D}^{\infty}\left\langle(y-D)_+\right\rangle dy=\int_{\mu_D}^{\infty}(D-\mu_D)^2\, g_D(D)dD$ (39)

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Summing (38) and (39)

$$2\int_{-\infty}^{\mu_D}\left\langle (y-D)_+\right\rangle dy+2\int_{\mu_D}^{\infty}\left\langle (y-D)_+\right\rangle dy$$

$$=\left[\int_{-\infty}^{\mu_D}(D-\mu_D)^2 g_D(D)dD+\int_{\mu_D}^{\infty}(D-\mu_D)^2 g_D(D)dD\right] \quad (40)$$

$$2\int_{-\infty}^{\infty}\left\langle (y-D)_+\right\rangle dy=\left[\int_{-\infty}^{\mu_D}(D-\mu_D)^2 g_D(D)dD+\int_{\mu_D}^{\infty}(D-\mu_D)^2 g_D(D)dD\right] \quad (41)$$

$$2\int_{-\infty}^{\infty}\left\langle (y-D)_+\right\rangle dy=$$

$$\left[\int_{-\infty}^{\mu_D}(D-\mu_D)^2\sum_{D_i\in\Omega_D}P(D_i)\delta(D-D_i)dD+\int_{\mu_D}^{\infty}(D-\mu_D)^2\sum_{D_i\in\Omega_D}P(D_i)\delta(D-D_i)dD\right] \quad (42)$$

$$2\int_{-\infty}^{\infty}\left\langle (y-D)_+\right\rangle dy=$$

$$\left[\sum_{D_i\in\Omega_D}P(D_i)\int_{-\infty}^{\mu_D}(D-\mu_D)^2\delta(D-D_i)dD+\sum_{D_i\in\Omega_D}P(D_i)\int_{\mu_D}^{\infty}(D-\mu_D)^2\delta(D-D_i)dD\right] \quad (43)$$

$$2\int_{-\infty}^{\infty}\left\langle (y-D)_+\right\rangle dy=\left[\sum_{D_i\in\Omega_D}P(D_i)(-D_i-\mu_D)^2+\sum_{D_i\in\Omega_D}P(D_i)(D_i-\mu_D)^2\right] \quad (44)$$

$$2\int_{-\infty}^{\infty}\left\langle (y-D)_+\right\rangle dy=\left[\sum_{D_i\in\Omega_D}P(D_i)(D_i+\mu_D)^2+\sum_{D_i\in\Omega_D}P(D_i)(D_i-\mu_D)^2\right] \quad (45)$$

Applying (50) below to the L.H.S of (41), we have

$$2\int_{-\infty}^{\infty}\left[\int_{0}^{\infty}\Pr(Y>D+y)\mathrm{dy}\right]dy=\int_{-\infty}^{\mu_D}(D-\mu_D)^2 g_D(D)dD+\int_{\mu_D}^{\infty}(D-\mu_D)^2 g_D(D)dD \quad (46)$$

Considering an excess of loss re-insurance on which a retention D is defined, then

$$Min(y-D,0)=(y\wedge D)\Rightarrow E(Min(y-D,0))=E(y\wedge D) \quad (46a)$$

Equivalently, $E(y\wedge D)=\int_{y=0}^{D} yg_Y(y)dy+dS_Y(y)=\int_{y=0}^{D} S_Y(y)dy$ (46b)

By definition, $Var(Min(y-D))=E\left((y\wedge D)^2\right)$ (46c)

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$$Var\left(Min\left(y-D\right)\right)=\int_{y=0}^{D} y^2 g_Y\left(y\right)dy+D^2S_Y\left(y\right)=2\int_{y=0}^{D} yS_Y\left(y\right)dy \quad (46d)$$

In (Schlesinger, 1981; Jack M., & Ormiston; 1999; Tse, 2009; Thogersen, 2016; Woodard & Yi 2018), we observe that with deductible modifications, the scheme holder will be responsible for the first D monetary units of each claim amount, although Woodard & Yi (2018) further estimated the insurance deductible under endogenous premium conditions using the calculus of utility function.
For instance, under a first party insurance cover, the underwriter will indemnify the scheme holder for the amount of severity exceeding the deductible as against the third party liability contract where the underwriter is liable to pay full severity amount, but the policy holder pays the underwriter for the fraction of the loss within the deductible condition. However, drawing inference from (Tse, 2009; Thogersen, 2016), it is possible that a scheme holder mitigates the policy cost if the insured peril is better than the mean loss in the rating categories of the insurance firm. Suppose severities do not include deductible information, then it will not be necessary to make any assumption about the underlying scheme. The profile of policy limits could generate assumptions on the distributions of the underlying scheme. We recall in (Tse, 2009; Ogungbenle et al, 2020) that the amount of claim size in the loss event L is defined by

$$Y_{\mathrm{L}}=\begin{cases}0, \mathrm{Y}\le D\\ Y-D, \mathrm{Y}>D\end{cases} \quad (47)$$

$$Y_{\mathrm{L}}=\left(Y-D\right)_{+} \quad (48)$$

where $Y_{+}=\begin{cases}0, \mathrm{Y}\le 0\\ \mathrm{Y}, \mathrm{Y}>0\end{cases}$ (49)

$$\left\langle Y_{\mathrm{L}}\right\rangle=\left\langle\left(Y-D\right)_{+}\right\rangle=\int_{0}^{\infty}\Pr\left(Y>D+y\right)\mathrm{dy}\le\int_{0}^{\infty}\frac{\left\langle Y\right\rangle}{\left(D+y\right)}\mathrm{dy}, \text{ by Markov's inequality} \quad (50)$$

$$\Pr\left(Y_{\mathrm{L}}=0\right)=G_{\mathrm{Y}}\left(\mathrm{D}\right) \quad (51)$$

Following, (Tse, 2009), Y_{L} has a probability mass point at zero of $G_{\mathrm{Y}}\left(D\right)$

$g_{Y_{\mathrm{L}}}\left(y\right)=g_{\mathrm{Y}}\left(y+D\right), y>0$ by definition of coverage modification (52)

The expected value function allows us to assess which losses from the risks, the insurance firm will bear in quantitative terms, using definition in equation (24), we observe that

$$\langle Y_L \rangle = \int_D^{\infty} (y - D)\, g_Y(y)\, dy \tag{53}$$

$$\langle Y_{\mathrm{L}} \rangle = \int_{-\infty}^{\infty} (Y - D)\, g_{Y_{\mathrm{L}}}(y) \mathrm{dy} = \int_0^{\infty} (Y - D)\, g_{Y_{\mathrm{L}}}(y) \mathrm{dy} \tag{54}$$

$$\langle (Y - D)_+ \rangle = \int_0^{\infty} \Pr(Y > D + y) g_{Y_{\mathrm{L}}}(y)\, \mathrm{dy} \tag{55}$$

$$\langle Y_{\mathrm{L}} \rangle = \int_D^{\infty} (Y - D)\, g_{Y_{\mathrm{L}}}(y) \mathrm{dy} = \langle (Y - D)_+ \rangle = \int_0^{\infty} \Pr(Y > D + y) g_{Y_{\mathrm{L}}}(y)\, \mathrm{dy} \tag{56}$$

It is observed in (Tse, 2009; Ogungbenle et al, 2020) that the indemnity function $f(D)$

$$f(D) = \langle (Y - D)_+ \rangle = \max((Y - D), 0) \tag{57}$$

and $\Pr((Y - D)_+ > y) = \Pr(Y > y + D)$ (58)

Using dirac-delta kernel, we obtain the first moment by equations (17)-(19)

$$\langle Y_{\mathrm{L}} \rangle = \int_{\mathrm{D}}^{\infty} (Y - D) \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} \delta(Y - y_{\mathrm{j}}^*) \mathrm{dy} = \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} \int_{\mathrm{D}}^{\infty} (Y - D) \delta(Y - y_{\mathrm{j}}^*) \mathrm{dy} \tag{59}$$

$$\langle Y_{\mathrm{L}} \rangle = \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} (y_{\mathrm{j}}^* - D) = \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} y_{\mathrm{j}}^* - \mathrm{D} \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} \tag{60}$$

$$\langle Y_{\mathrm{L}} \rangle = \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} y_{\mathrm{j}}^* - \mathrm{D} \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j}\text{, recall that } \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} = 1 \tag{61}$$

$$\langle Y_{\mathrm{L}} \rangle = \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{P_j} y_{\mathrm{j}}^* - D \tag{62}$$

$$\langle Y_{\mathrm{L}} - \langle Y_{\mathrm{L}} \rangle \rangle = \int_{-\infty}^{\infty} (Y_{\mathrm{L}} - \langle Y_{\mathrm{L}} \rangle) dG_Y(y) = \int_{-\infty}^{\infty} Y_{\mathrm{L}}\, dG_Y(y) - \int_{-\infty}^{\infty} \langle Y_{\mathrm{L}} \rangle dG_Y(y) =$$

$$\int_{-\infty}^{\infty} Y_{\mathrm{L}}\, dG_Y(y) - \langle Y_{\mathrm{L}} \rangle \int_{-\infty}^{\infty} g_Y(y) dy = \langle Y_{\mathrm{L}} \rangle - \langle Y_{\mathrm{L}} \rangle = 0 \quad , \tag{63}$$

3.4 Theorem

If $Y_{\mathrm{L}} = (Y - D)_+$ then, $E(Y - D)_+ = \int_0^{\infty} \Pr(Y > D + y) \mathrm{dy}$

Proof

$$\int_{D}^{\infty}\left(1-G_Y(u)\right)\mathrm{du}=\int_{D}^{\infty}\Pr\left(Y>u\right)\mathrm{du} \tag{64}$$

$$\int_{D}^{\infty}\Pr\left(Y>u\right)\mathrm{du}=\int_{D}^{\infty}\int_{0}^{\infty}I_{(u,\infty)}\left(Y\right)dG_Y\left(y\right)\mathrm{du} \tag{65}$$

$$\int_{D}^{\infty}\Pr\left(Y>u\right)\mathrm{du}=\int_{0}^{\infty}\int_{D}^{\infty}I_{(0,y)}\left(u\right)dudG_Y\left(y\right) \tag{66}$$

Where $I_{(.)}$ is the indicator function

$$\int_{D}^{\infty}\Pr\left(Y>u\right)\mathrm{du}=\int_{0}^{\infty}\left(y-D\right)_{+}dG_Y\left(y\right) \tag{67}$$

$$\int_{D}^{\infty}\Pr\left(Y>u\right)\mathrm{du}=E\left(Y-D\right)_{+}=\max\left(\left(Y-D\right),0\right) \tag{68}$$

The expectation in equation (68) implies

$$\left\langle Y_{\mathrm{L}}\right\rangle=\left\langle\left(Y-D\right)_{+}\right\rangle=\int_{0}^{\infty}\Pr\left(Y>D+y\right)\mathrm{dy} \tag{69}$$

and the result follows. In view of the identity (70) stated below and equation (69)

$$E\left(Y\right)\equiv\left\langle\left(Y-D\right)_{+}\right\rangle-\left\langle\left(D-Y\right)_{+}\right\rangle+D \tag{70}$$

Where $\left\langle\left(D-Y\right)_{+}\right\rangle=\int_{-\infty}^{D}G_Y\left(y\right)dy$ (71)

hence $\int_{D}^{\infty}\Pr\left(Y>D+y\right)\mathrm{dy}-\int_{-\infty}^{D}G_Y\left(y\right)dy=\left\langle Y\right\rangle-D$ (72)

4. RESULTS

Cauchy-Bunyakovsky- Schwarz inequality in constructing upper bound for cost per loss payment.

4.1 Theorem

$$\int_{D}^{\infty}\left(y-D\right)g_Y\left(y\right)dy\le\frac{\left(\left\langle y^2\right\rangle-2D\left\langle y\right\rangle+D^2\right)^{0.5}+\left(\left\langle y\right\rangle-D\right)}{2}$$

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Proof

We use the results in the previous theorems as part of the proof

$$\langle Y_L \rangle = \int_D^{\infty} (y-D)\, g_Y(y)\, dy \tag{73}$$

$$\langle Y_L \rangle_2 = \int_{-\omega}^{D} (y-D)\, g_Y(y)\, dy, \langle Y_L \rangle_1 = \int_D^{\infty} (y-D)\, g_Y(y)\, dy,\ for, 0<\omega<D \tag{74}$$

$$\langle Y_L \rangle_1 + \langle Y_L \rangle_2 = \int_{-\omega}^{\infty} (y-D)\, g_Y(y)\, dy = \left(\langle y \rangle - D\right), \omega \varepsilon R^+ \tag{75}$$

$$\langle Y_L \rangle_1 - \langle Y_L \rangle_2 = \int_D^{\infty} (y-D)\, dG_Y(y) - \int_{-\omega}^{D} (y-D)\, dG_Y(y) \tag{76}$$

$$\langle Y_L \rangle_1 - \langle Y_L \rangle_2 = \int_{-\omega}^{\infty} \left|(y-D)\right| dG_Y(y) \tag{77}$$

$$\left[\langle Y_L \rangle_1 - \langle Y_L \rangle_2\right]^2 = \left[\int_{-\omega}^{\infty} \left|(y-D)\right| dG_Y(y)\right]^2 \tag{78}$$

By the Cauchy-Bunyakovsky-Schwarz inequality

$$\left[\int_{-\omega}^{\infty} \left|(y-D)\right| dG_Y(y)\right]^2 \le \int_{-\omega}^{\infty} dG_Y(y) \int_{-\omega}^{\infty} (y-D)^2\, dG_Y(y) = \left(G_Y(\infty) - G_Y(-\infty)\right) \int_{-\omega}^{\infty} (y-D)^2\, dG_Y(y) \tag{79}$$

$$\left[\int_{-\omega}^{\infty} \left|(y-D)\right| dG_Y(y)\right]^2 \le \int_{-\omega}^{\infty} dG_Y(y) \int_{-\omega}^{\infty} (y-D)^2\, dG_Y(y) = \left(G_Y(\infty) - G_Y(-\omega)\right) \int_{-\omega}^{\infty} (y-D)^2\, dG_Y(y) = (1-0) \int_{-\omega}^{\infty} (y-D)^2\, dG_Y(y) \tag{80}$$

$$\left[\int_{-\omega}^{\infty} \left|(y-D)\right| dG_Y(y)\right]^2 \le \int_{-\omega}^{\infty} \left(y^2 - 2yD + D^2\right) dG_Y(y) \tag{81}$$

$$\left[\int_{-\omega}^{\infty} \left|(y-D)\right| dG_Y(y)\right]^2 \le \int_{-\omega}^{\infty} \left(y^2 - 2yD + D^2\right) g_Y(y)\, dy = \left(\langle y^2 \rangle - 2D\langle y \rangle + D^2\right) \tag{82}$$

$$\langle Y_L \rangle_1 - \langle Y_L \rangle_2 \le \left(\langle y^2 \rangle - 2D\mu_Y + D^2\right)^{0.5} \text{ and } \langle Y_L \rangle_1 + \langle Y_L \rangle_2 = \left(\langle y \rangle - D\right) \tag{83}$$

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Solving equations (83) simultaneously, we have

$$\langle Y_L \rangle_1 = \int_D^{\infty} (y-D)\, g_Y(y)\, dy \le \frac{\left(\langle y^2 \rangle - 2D\langle y \rangle + D^2\right)^{0.5} + \left(\langle y \rangle - D\right)}{2} \tag{84}$$

Since an insurance policy is a contract, the insurance firm cannot just repudiate claim on the grounds of computing premium incorrectly. It is observed in (Schlesinger, 1985; Tse, 2009) that an insurance contract is a guarantee by an underwriter that it will indemnify for losses incurred by insured perils. Underwriting assumes the risk that the premium may be insufficient to pay claims and expenses. However, the insurer must complement this risk by keeping adequate capital to enable it have certain level of expected return on capital. By writing new insurance contracts, the insurance firm collects premiums and invests the proceeds to generate profit. Furthermore, in view of (Tse, 2009; Sakthivel & Rajitha 2017), a large severity claim will be more expensive than an average claim but a small severity claim will be less expensive than the average claim. We also recall in (Tse, 2009; Ogungbenle et al, 2020) that

$$\langle Y_L^2 \rangle = \int_{-\infty}^{\infty} (y-D)^2\, g_{Y_L}(y)\, \mathrm{dy} \tag{85}$$

Since density is only defined on the real line, we integrate from zero to infinity

$$\langle Y_L^2 \rangle = \int_0^{\infty} (y-D)^2\, g_{Y_L}(y)\, \mathrm{dy} \tag{86}$$

But by the definition of deductible, we integrate from D to infinity

$$\langle Y_L^2 \rangle = \int_D^{\infty} (y-D)^2 \sum_{j=1}^{m} P_j \delta\left(y - y_j^*\right) \mathrm{dy} \tag{87}$$

$$\langle Y_L^2 \rangle = \sum_{j=1}^{m} P_j \int_D^{\infty} (y-D)^2\, \delta\left(y - y_j^*\right) \mathrm{dy} \tag{88}$$

$$\langle Y_L^2 \rangle = \sum_{j=1}^{m} P_j \left(y_j^* - D\right)^2 = \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - 2D\sum_{j=1}^{m} P_j y_j^* + \sum_{j=1}^{m} P_j D^2 \tag{89}$$

$$\langle Y_L^2 \rangle = \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - 2D\sum_{j=1}^{m} P_j y_j^* + D^2 \tag{90}$$

In view of (Tse, 2009; Ogungbenle et al, 2020)

$$V(Y_L) = \langle Y_L^2 \rangle - \langle Y_L \rangle^2 \tag{91}$$

$$V(Y_L) = \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - 2D\sum_{j=1}^{m} P_j y_j^* + \sum_{j=1}^{m} P_j D^2 - \left(\sum_{j=1}^{m} P_j y_j^* - \sum_{j=1}^{m} DP_j\right)^2 \tag{92}$$

$$V(Y_L) = \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - 2D\sum_{j=1}^{m} P_j y_j^* + \sum_{j=1}^{m} P_j D^2 - \left(\sum_{j=1}^{m} P_j y_j^*\right)^2 +$$

$$2\sum_{j=1}^{m} P_j y_j^* \sum_{j=1}^{m} DP_j - \left(\sum_{j=1}^{m} DP_j\right)^2 \tag{93}$$

$$V(Y_L) = \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - \left(\sum_{j=1}^{m} P_j y_j^*\right)^2 + 2\sum_{j=1}^{m} P_j y_j^* \sum_{j=1}^{m} DP_j - 2D\sum_{j=1}^{m} P_j y_j^*$$

$$+\sum_{j=1}^{m} P_j D^2 - \left(\sum_{j=1}^{m} DP_j\right)^2 \tag{94}$$

$$V(Y_L) = \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - \left(\sum_{j=1}^{m} P_j y_j^*\right)^2 + 2D\sum_{j=1}^{m} P_j y_j^* - 2D\sum_{j=1}^{m} P_j y_j^* + D^2 - D^2$$

$$= \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - \left(\sum_{j=1}^{m} P_j y_j^*\right)^2 \tag{95}$$

$$V_y^* = \sum_{j=1}^{m} P_j \left(y_j^*\right)^2 - \left(\sum_{j=1}^{m} P_j y_j^*\right)^2 \tag{96}$$

4.2 Theorem

If $\zeta(D)$ is a differentiable function of premium D

with $\frac{\zeta}{1+\beta} = E\left(\Lambda(L)\right)$ (97)

where $\Lambda(L)$ is the maximum loss and the loading is given by

$\psi = 1+\beta$, then the density is $g_{Y_L}(D) = \frac{1}{(1+\beta)} \frac{\partial \zeta^2}{\partial D^2}$ (98)

Proof

Recall in (Tse, 2009; Ogungbenle et al, 2020), the expected indemnity is the ratio of the premium due to the loading factor. Since underwriting risk exists, underwriters will cover such risk through the introduction of security loading ψ in computing the premium.

$\psi = (1+\beta)$ and hence

$\zeta = E\left(\Lambda(L)\right)(1+\beta)$ where $\zeta(D)$ is the premium chargeable and a differentiable function of D.

$(1+\beta)$ is the safety loading which guides against larger than anticipated loss and $\Lambda(L)$ is the maximum loss $0 < \Lambda(L) < \bar{y}$

$\zeta = \psi \times E(\Lambda(L))$ implies

$$\zeta = \psi \times \int_D^{\bar{y}} (y-D)\, dG_{Y_L}(y) = \int_D^{\bar{y}} (1+\beta)(y-D)\, g_{Y_L}(y)\, dy,\ \bar{y} < n_0 \tag{99}$$

$$\zeta = \psi \times \int_D^{\bar{y}} (y-D)\, dG_{Y_L}(y) \cong \int_D^{\bar{y}} (1+\beta)(y-D) \sum_{j=1}^{m} P_j \delta\left(y - y_j^*\right) dy \tag{100}$$

$$\zeta = \psi P_j \sum_{j=1}^{m} \int_D^{\bar{y}} \delta\left(y - y_j^*\right)(y-D) \cong (1+\beta) P_j \sum_{j=1}^{m} \left(y_j^* - D\right), D < y_j^* < \bar{y} \tag{101}$$

$$\zeta(s) = \int_{a(s)}^{b(s)} g(y,s)\, dy, \text{then, } \frac{d\zeta(s)}{ds} = \int_{a(s)}^{b(s)} \frac{\partial}{\partial s} g(y,s)\, dy + g(b(s),s) \frac{\partial}{\partial s} b(s) - g(a(s),s) \frac{\partial}{\partial s} a(s) \tag{102}$$

Differentiating *LHS* of equation (99) with respect to D by applying (102) we have

$$\frac{\partial \zeta}{\partial D} = -(1+\beta)\left(1 - g_{Y_L}(D)\right) = -(1+\beta)\left(G_{Y_L}(\bar{y}) - G_{Y_L}(D)\right) \tag{103}$$

$$\frac{\partial \zeta}{\partial D} = -(1+\beta)\left(G_{Y_L}(\bar{y}) - G_{Y_L}(D)\right) \tag{104}$$

$$\frac{\partial \zeta^2}{\partial D^2} = (1+\beta)\left(G'_{Y_L}(D)\right) = (1+\beta)\left(g_{Y_L}(D)\right) \tag{105}$$

$$g_{Y_L}(D) = \frac{1}{(1+\beta)} \frac{\partial \zeta^2}{\partial D^2} \tag{106}$$

$$\frac{\partial \zeta^3}{\partial D^3} = (1+\beta)\left(g'_{Y_L}(D)\right) \tag{107}$$

Thus, $\frac{\partial^3 \eta}{\partial D^3}$ is positive since D and ψ are greater than zero. We also note from equation (99) that

$$\frac{\partial \zeta^{(k)}}{\partial D^k} = (1+\beta)\left(g_{Y_L}^{(k-2)}(D)\right); k = 2,3,4,\ldots \tag{108}$$

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$$g'_{Y_L}(D) \cong \frac{1}{(1+\beta)}\frac{\partial\zeta^3}{\partial D^3};\beta \neq -1 \tag{109}$$

5. DISCUSSION OF RESULTS

We recall equation (84) that

$$\langle Y_L\rangle_1 = \int_D^\infty (y-D)g_Y(y)dy \leq \frac{\left(\langle y^2\rangle - 2D\langle y\rangle + D^2\right)^{0.5} + \left(\langle y\rangle - D\right)}{2} \tag{110}$$

Substituting (70) in (84), we have

$$\langle Y_L\rangle_1 \leq \frac{\left(\langle y^2\rangle - 2D\langle y\rangle + D^2\right)^{0.5} + \left(\langle (Y-D)_+\rangle - \langle (D-Y)_+\rangle\right)}{2} \tag{111}$$

and by the results in equation (61)-(63)

$$\sum_{j=1}^{m} P_j y_j^* - D \leq \frac{\left(\langle y^2\rangle - 2D\langle y\rangle + D^2\right)^{0.5} + \left(\langle (Y-D)_+\rangle - \langle (D-Y)_+\rangle\right)}{2} \tag{112}$$

But by Milne's inequality,

$$\left(\sum_{j=1}^{m} P_j y_j^*\right)^2 \leq \left(\sum_{j=1}^{m} P_j^2 + \sum_{j=1}^{m}\left(y_j^*\right)^2\right)^2 \frac{\sum_{j=1}^{m} P_j^2 \sum_{j=1}^{m}\left(y_j^*\right)^2}{P_j^2 + \sum_{j=1}^{m}\left(y_j^*\right)^2} \leq \sum_{j=1}^{m} P_j^2 \sum_{j=1}^{m}\left(y_j^*\right)^2 \Rightarrow \tag{113}$$

$$\left(\sum_{j=1}^{m} P_j y_j^*\right) \leq \sqrt{\sum_{j=1}^{m} P_j^2 \sum_{j=1}^{m}\left(y_j^*\right)^2} \text{ and consequently, from equation (113)} \tag{114}$$

$$\sqrt{\sum_{j=1}^{m} P_j^2 \sum_{j=1}^{m} y_j^{*2}} - D \leq \frac{\left(\langle y^2\rangle - 2D\langle y\rangle + D^2\right)^{0.5} + \left(\langle (Y-D)_+\rangle - \langle (D-Y)_+\rangle\right)}{2}. \tag{115}$$

However if P_j and y_j^* are directly proportional, then $P_j = Dy_j^*$ for real D and equality holds.

From the analytical point of Cauchy's inequality, where new insurance markets are poorly explored, then bounds can be constructed to generate an estimate of the probable cost of the insurance product applying risk data. The Cauchy's inequality has provided a natural generalization for the actuarial risk functions as applicable to severities. Evolution of financial engineering in actuarial applications could be crucial in appraising underwriting performance by paying significant attention to regulatory capital and solvency requirements. Actuaries analyze the underwriting risk profile of the potential scheme holder and approximate the probability that a scheme holder experiences loss and to what level and based on this profile, premium could be obtained. Underwriting risk governed by the relation

$$\int_R premium + \int_R Risk\ capital \leq \int_R claims \tag{116}$$

occurs mainly as a result of extreme claims and besetting management expenses. Underwriting loss could also occur as a result of inordinate assessment of the loss connected with underwriting a scheme or from chaotic economic conditions and consequently, the insurer's costs may significantly exceed earned premiums. Mean loss explains severity incurred after an insurance firm has paid out claims or when it has to pay out more claims than expected such that the premiums collected could not cover the total expenses. The loss incurred technically shows the inefficiency of the firm's underwriting capabilities. Obtaining adequate premiums is very complex since each scheme holder has a unique loss profile.

6. CONCLUSION

In the current paper, the opportunities for modeling and estimating claim severity was extended to upper bound. The rationale behind the estimation of upper bound for the loss distribution implies an extension of Cauchy-Bunyakovsky-Schwartz inequality. The Cauchy-Bunyakovsky-Schwartz inequality has been successfully applied in this paper the goal of which is to obtain upper bound for computing the expected claim per loss payment in general insurance practice**.** Bearing that it is sufficient for underwriters to arrive at adequate premium, it is just as equally sufficient to obtain the mean severities analytically or numerically. Future research direction may carry out simulation of the scheme used here.

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