SAMA CIRCULAR MODEL

The Sama Circular Model was introduced in year 2018. It was the improved version of the Circular Model of W. G. Samanthi Konarasinghe (2016), Sri Lanka. Development of the CM was based on theory of Uniform Circular motion, Fourier Transformation and Least Square Regression Analysis. Fourier transformation (FT) can be used to transform a real valued function f(x) into series of trigonometric functions. FT has two versions; discrete transformation and continuous transformation.

The discrete version of Fourier transformation is; $f(x) = \sum_{-\infty}^{\infty} a_n e^{-k\theta}$

According to De Moivre's theorem; $e^{-k\theta} = \cos k\theta + i \sin k\theta$

Where, i is a complex number. Therefore *f(x)* can be written as:

$$f_x = \sum_{k=1}^{n} a_k \cos k\theta + b_k \sin k\theta \qquad (1)$$

Whare a_k and b_k are amplitudes, k is the harmonic of oscillation.

A particle *P,* which is moving in a horizontal circle of centre O and radius a is given in Figure 1, ω is the angular speed of the particle;

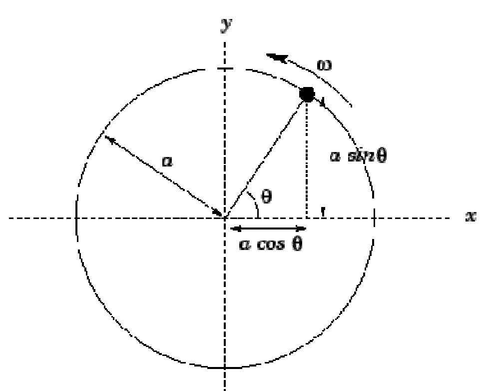

Figure 1- Motion of a particle in a horizontal circle

Angular speed is defined as the rate of change of the angle with respect to time. Then;

$$\omega = \frac{d\theta}{dt}$$

$$\int_0^\theta d\theta = \int_0^{\mathrm{t}} \omega \,\mathrm{dt}$$

Hence, $\theta = \omega t$ (2)

Substitute (2) in (1); $f_x = \sum_{k=1}^{n} a_k \cos k\omega t + b_k \sin k\omega t$ (3)

At one complete circle θ=2π radians. Therefore, the time taken for one complete circle (*T*) is given by: $T = 2\pi / \omega$ (4)

In circular motion, the time taken for one complete circle is known as the period of oscillation. In other words, the period of oscillation is equal to the time between two peaks or troughs of sine or cosine function. If a time series follows a wave with *f* peaks in *N* observations, its period of oscillation can be given as;

$$T = \frac{\textit{total}\text{ number of periods}}{\text{total number of peaks}} = \frac{N}{f} \tag{5}$$

Equating (4) and (5); $\dfrac{2\pi}{\omega} = \dfrac{N}{f}$

Then, $\omega = 2\pi \dfrac{f}{N}$

However, (1) it is a deterministic model, does not capture the randomness in real life. Therefore (1) is modified as follows;

$$Y_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \qquad (6)$$

The model (6) was named as "Circular Model".

Y_t is a continuous random variable

$t \geq 0,$

$k \in Z^+$

Series, sin $k\omega t$ cos $k\omega t$ are independent

ε is Normally distributed d with 0 mean and constant variance

ε is Independent

The CM cannot be applied, if Y_t has a wave like pattern with trend. Samanthi Konarasinghe applied the method of differencing to mitigate the limitation of the CM. In usual notation, differencing series of Y_t are as follows;

First differenced series: $Y_t^{'} = Y_t - Y_{t-1} = (1-B)Y_t$ (7)

Second differenced series:

$$Y_t^{''} = Y_t^{'} - Y_{t-1}^{'} = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2)}) = Y_t - 2Y_{t-1} + Y_{t-2} = (1-B)^2 Y_t \qquad (8)$$

Similarly, d^{th} order difference is, $Y_t^d = Y_t - Y_{t-d} = (1-B)^d Y_t$ (9)

Where, B is the Back Shift operator; $BY_t = Y_{t-1}$.

Assume Y_t^d is trend free. Let, $Y_t^d = X_t$. Then X_t could be modeled as;

$$X_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \tag{10}$$

Hence;

$$\begin{aligned} Y_t - Y_{t-d} &= \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \\ Y_t &= Y_{t-d} + \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \end{aligned} \tag{11}$$

The model (11); improved Circular Model, is named as **"Sama Circular Model (SCM)".** The SCM can be applied to model any wave like pattern; regular or irregular, with trend or trend free. In general time series data contains; Trend, Seasonal component and Cyclical component. The SCM is capable in capturing all the three components without de-trending or de-seasonalizing.