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Sama Radial Indicator (SRI) for Measuring Qualitative Variables

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ABSTRACT

Measuring qualitative variables, analyzing them and interpreting them are more challenging than their quantitative counterparts, especially when they are not directly observable. Such variables are captured through Latent Variables (LVs) or hidden variables. These hidden variables are measured by items (questions) and the item responses are obtained by dichotomous or polytomous item scales. The existing item scales have a limited number of response options. For example, the dichotomous scale has only two possible options, and the Likert scales have three, five or seven options. Therefore LV's are categorical types, it is hard to convert them into a continuous scale. Hence categorical data should be analyzed by non-parametric methods. However, the power of parametric tests is higher than the non-parametric counterparts, hence researchers tend to misuse statistical techniques in the analysis of LVs. This study aimed to develop a measurement scale for qualitative variables and mitigate the limitations in the data analysis. The newly developed measurement scale, named "Sama Radial Indicator (SRI)", converts the psychometric responses of the respondents into a continuous random variable, hence either the probability distribution or the sampling distribution of the LVs would meet the normality criteria. Therefore parametric statistical techniques are suitable for the data analysis. The SRI technique would solve a decades-old problem in various fields of research including, Psychology, Management, Business & Marketing, Medical & Healthcare, Education and more. It is recommended to test the efficiency of the SRI with real-life applications.

Keywords: Qualitative variable, Latent variable, Item scale.

1. INTRODUCTION

1.1 Background of the Study

Studies in the various fields of research involve qualitative variables. Some qualitative variables are not directly observed, hence are inferred from other, observable variables referred to as hidden variables or Latent Variables (LV) (Borsboom & Molenaar, 2015); (Ge et al., 2019). For example, researchers in the fields of psychology, education, business, marketing, medicine & healthcare etc., need to measure the attitudes, beliefs and traits of individuals. These qualitative variables are captured by LVs and the LVs are measured by multi-item scales where an “item” is a question, and a “scale” is the resulting estimate of the LV (Furr, 2011).

1.2 Research Problem

The Likert scales are widely applied polytomous rating scales in various fields of research. In general, the Likert scale uses three, five or seven comparable groups to obtain the item response for a latent variable (Likert, 1932). These variables are categorical types, measured by the ordinal measurement scale. In other words, measurements of these variables are comparable, but the mathematical operations would not yield any meaningful outcome. Therefore assuming the Likert-type categories constitute interval-level measurement is incorrect (Knapp, 1990); (Kuzon et al., 1996); (Susan, 2004); (Allen & Seaman, 2007). Categorical data should not be analyzed by the parametric methods unless they meet the normality criteria. For example: mean and variance are not suitable measurements for categorical variables; multiple regression analysis cannot be performed with categorical response variables; Z-test or ANOVA are not appropriate for comparison of categorical variables (Attwood et al., 2009); (Allen & Seaman, 2007); (Carifio & Perla, 2008).

Variables measured by polytomous item scales, including the Likert scales are very unlikely to be normally distributed (Susan, 2004); (Allen & Seaman, 2007), but it has become a common practice to assume that Likert-type scales are interval scales, hence measurement become normally distributed. Under these false assumptions, researchers use parametric methods to analyse categorical variables and come to meaningless conclusions. This may have become a practice as parametric methods are more statistically powerful than their non-parametric counterparts (Carifio & Perla, 2008); (Bishop, & Herron, 2015). Developing a suitable item scale to mitigate the limitations of polytomous rating scales would be the solution to this problem. Hence this study aimed to fill the knowledge gap.

1.3 Objective of the Study

To develop a suitable measurement scale to measure the qualitative variables.

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1.4 Significance of the Study

Misuse of statistics has become a serious problem in research. Among them, the misuse of Likert-type polytomous scales is common all over the world. This happens due to the negligence of researchers as well as some limitations of the existing polytomous rating scales. This study will introduce an effective item scale to measure qualitative variables, which would help to solve the decades-old problem in various fields of research.

2. LITERATURE REVIEW

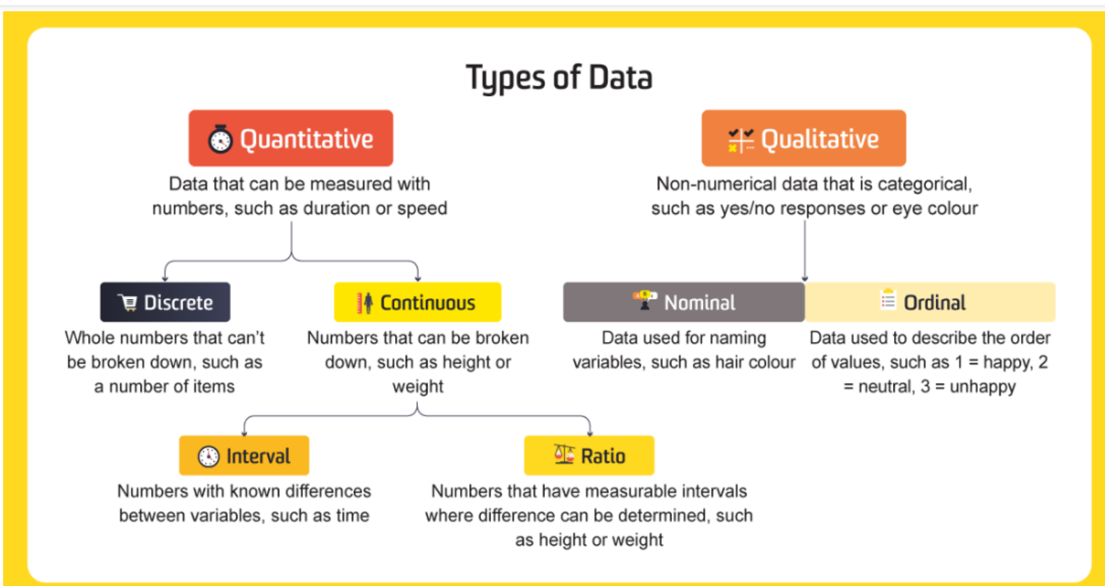
The literature review consists three parts:

2.1 Types of random variables, measurement scales and probability distributions
2.2 Likert scale
2.3 Properties of an efficient measurement scale

2.1 Types of Random Variables, Measurement Scales and Probability Distributions

A random variable is an outcome from a probability experiment, so its value is determined by chance. Types of random variables and corresponding measurement scales are shown in Figure 1 (UNSW Sydney Online, 2022).

Figure 1: Types of Data



Source: https://studyonline.unsw.edu.au/blog/types-of-data

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Random Variables are mainly classified into two parts: quantitative and qualitative. Variables associated with measurements are called quantitative variables whilst variables not associated with measurements are called qualitative variables.
Quantitative variables are measured by “interval” or “ratio” scale and qualitative variables are measured by “nominal” or “ordinal” scale. For example, the temperature of different cities of a country is a quantitative variable measured by the interval scale; the heights of students in a school is a quantitative variable measured by the ratio scale; the gender of a person is a qualitative variable measured by the nominal scale and the level of educational qualifications of people in a country (secondary school, high school, diploma, degree, etc.) is a qualitative variable measured by the ordinal scale.

A probability distribution provides the possible values of the random variable and its corresponding probabilities. A probability distribution associated with a discrete random variable is known as a discrete probability distribution whilst a probability distribution associated with a continuous random variable is known as a continuous probability distribution. The distributions: Bernoulli, Binomial, Poisson, and Geometric are examples for discrete distributions and the distributions: Normal, Chi-square, and Student-t are examples for continuous probability distributions.

2.2 Likert Scale

The Likert scale assigns “codes or ranks” (numerical values) to categories of a qualitative variable (Likert, 1932). Several items were developed to measure a latent variable (LV) and then the total of those item ranks was taken as the outcome of the random variable. The method, named, "Sigma Method of Scoring" was tested with a sample of 100 respondents using 12, 15 and 24 items for each LV and found that the rank summations were approximately Normally Distributed (Likert, 1932). Hence, the study assumed that, if rank totals of LVs are measured by several items and rank totals are taken as the outcome, then they would be approximately normally distributed. However, the study has mentioned the danger of this assumption and emphasis the importance of proving the correctness or incorrectness (Likert, 1932).

Trace lines from graded model for 5- point Likert-scale responses to an attitude item is shown in Figure 2. The distribution of some responses are symmetrical, but others are non- symmetrical. Hence, this type of variables follow Logistic Distribution, not the Normal Distribution (Samejima, 1969); (Thissen, 1983).

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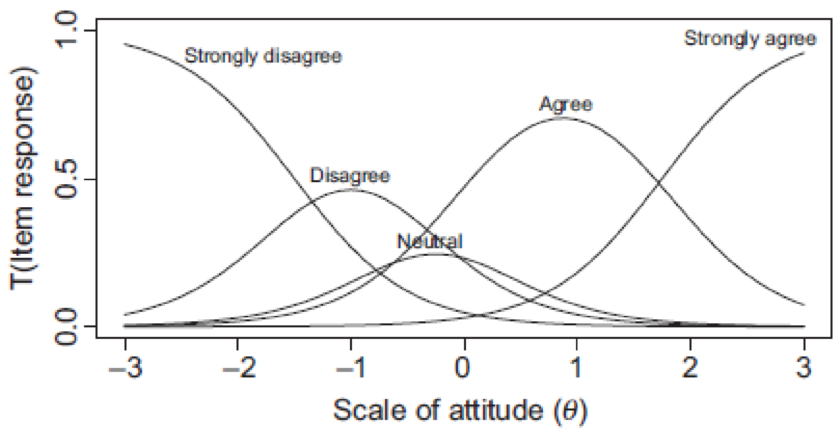
Figure 2: Scale Lines (Source: Samejima's 1969)



If 10-25 items are used to measure one LV and rank summation is taken as the outcome of the variable, then the variable may be approximately normally distributed (Likert, 1932), but the inclusion of a large number of items leads to higher non-response rate and many other problems (Paulhus, 1991).

Knapp (1990), Kuzon et al. (1996), Susan (2004) and Allen and Seaman (2007) have clearly explained that the response categories in Likert scales have a rank order, but the intervals between values cannot be presumed equal. Therefore assuming the Likert-type categories constitute interval-level measurement is incorrect. Susan (2004) named this misuse as Ab(using) the Likert scale and Kuzon et al. (1996) consider it a deadly sin of statistical analysis.

2.3 Properties of an Efficient Measurement Scale

An efficient measurement scale should contain higher response options, but each response should be differentiated from the others (Furr, 2011); (Diener et al., 1985); (Allen & Seaman, 2007); (Thissen, 2015). For example, a response scale including: "strongly agree, moderately agree, slightly agree, neutral, slightly disagree, disagree, strongly disagree", has higher response options compared to a dichotomous or five-point scale, but the respondent might not understand the difference between moderately agree and slightly agree and vice versa.

Having a neutral mid-point is another property of an efficient scale, as it would be the right response for some respondents. For example, a four-point scale including, "strongly agree, agree, disagree, strongly disagree", does not allow a respondent to neither agree nor disagree (Furr, 2011).

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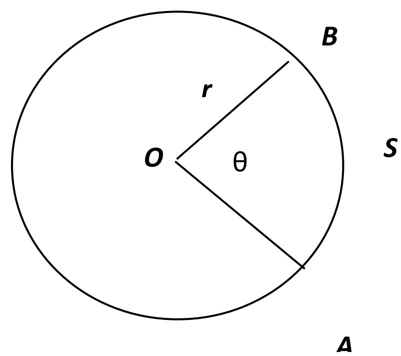
3. METHODOLOGY

This study uses the fundamentals of pure mathematics to develop an efficient item scale to overcome the limitations of existing measurement scales.

Consider a circle with center *O* and the radius r. The AB is an arc subtended an angle θ (radians) at the centre *O*;

Figure 3: Length of an Arc



$$\frac{\text{Length of arc}}{\text{Circumference}} = \frac{\text{Angle AOB}}{\text{Total angle around O}}$$

$$\frac{S}{2\pi r} = \frac{\theta}{2\pi}$$

$$\text{Hence, } S = r\theta$$

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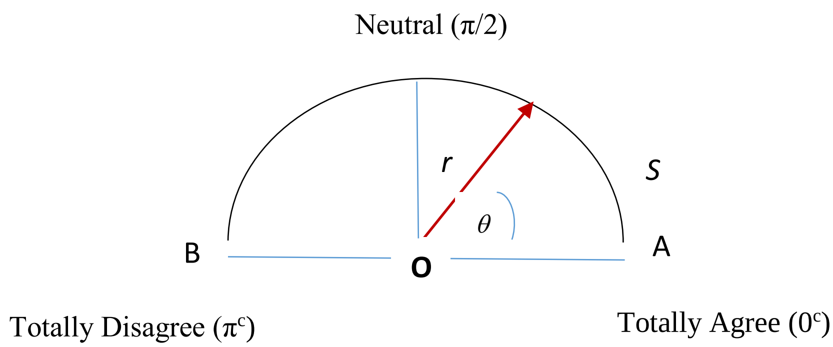
4. RESULTS

The proposed measurement scale is a semi-circle and the item response is an arc of the semi-circle (Figure 4).

Figure 4: Sama Radial Indicator (SRI)



In the usual notations, AP is an arc with length ***S***;

$$S = r\theta$$

Let, $r = 100$

Then, $S = 100\theta$

Item Response $(S): 0 \leq S \leq 100\pi$

$$0 \leq S \leq 314.16$$

Where, S is a continuous random variable.

A table of arc lengths (SRI Table**)** is developed to obtain the value of the item response of an individual. A part of the SRI Table is as follows;

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Table 1: SRI Table

θ^0	S	Interpretation
0	0	$\theta = 0$: Totally Agree
1	1.74603	
2	3.49206	$0 < \theta \leq 45$: Strongly Agree
……	………	
45	78.5714	$46 < \theta < 90$: Weakly Agree
…….	……….	
90	157.143	$\theta = 90$: Neutral
……..	……….	$91 < \theta < 135$: Weakly Disagree
……..	………	
135	235.714	$135 \leq \theta < 180$: Strongly Disagree
………	………	
……….	………	
180	314.286	$\theta = 180$: Totally Disagree

5. CONCLUSION AND RECOMMANDATIONS

The study is focused to develop an effective measurement scale to measure the qualitative random variables. The Sama Radial Indicator (SRI) gives 180 response options to the respondents and the outcome of the respondent will become a continuous random variable. Hence, it is concluded that the probability distribution of it will be normally distributed or sampling distribution will be approximately normally distributed.

This indicator shows only three points with verbal expressions: totally agree, neutral and totally disagree", hence the respondent would not get confuse with the meaning of responses. The new measurement scale has a higher response options and each response can be differentiated from the others, as explained in Table 1. The interpretations given in the SRI table help the researcher to understand the responses.

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It is recommended to use the SRI for collecting real-life data to check the efficiency of it.

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