MODEL DEVELOPMENTS FOR STOCK RETURNS

by

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DECLARATION

I do hereby declare that the thesis was exclusively carried out by me under the supervision of Dr. (Mrs.) N. R Abeynayake, Senior Lecturer, Faculty of Agriculture and Plantation Management, Wayamba University of Sri Lanka, and Dr. L. H. P. Gunaratne, Professor, Department of Agricultural Economics and Business Management, Faculty of Agriculture, University of Peradeniya, Sri Lanka. It describes the results of my own independent research except where due reference has been made in the text. No part of this thesis has been submitted earlier or concurrently for the same or any other degree.

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ABSTRACT

Statistical modeling is vital in planning, forecasting and management in the fields of; Agriculture, Medicine, Engineering, Economics, Finance and many more. Stock market is the secondary capital market which gives investment opportunities to investors and liquidity to firms. Share trading affects all the stake holders; customers of the firms, employees of the firms, the government and society. As such, share trading is an important part of the economy of a country. Share market investments are considered as high risk, high return investments, but the investors are concerned about high return at low risk. Hence forecasting of risk and return are essential for share markets.

The Capital Asset Pricing Model (CAPM) is the most commonly used model for forecasting share returns. However, CAPM has been subjected to empirical arguments over the decades. Literature gives enough evidence for the inefficiency of CAPM in forecasting returns. Yet, the Sri Lankan stock market depends on CAPM for forecasting risk and returns of securities. The study was focused to; test existing statistical models for forecasting Sri Lankan stock returns and develop suitable techniques for forecasting risk and returns.

Daily closing share price data, monthly sector indices and All Share Price Index data were obtained from Colombo Stock Exchange (CSE). Monthly share returns of a random sample of companies from CSE were used for data analysis. Pattern recognition of returns was the first step of the study. It was found that the Sri Lankan stock returns are stationary type, follow wave like patterns. Then the covariance structure of Sri Lankan

stock returns was identified. The central assertion of CAPM; linear relationship between individual company returns and the total market returns were not supported by the analysis, but there was a significant linear relationship between individual company returns and corresponding sector returns. Therefore individual company returns were modeled on sector returns. Thereafter, Auto Regressive Integrated Moving Average (ARIMA) models were tested on individual company returns. Finally, a technique based on Fourier transformation was applied on returns. The suggested model was named as the "Circular Model (CM)". Goodness of fit of the models was tested by residual plots, Auto Correlation Functions and Partial Autocorrelation Functions of residuals, Ljung-Box Q statistics, Durbin Watson test statistic and Anderson Darling test. Forecasting errors were measured by the Root Mean Square Error and the Mean Absolute Deviation. The ARIMA and CM were successful in forecasting returns. However, pattern of ARIMA forecasts was not close to the patterns of actual returns, while CM forecasts followed the actual returns. It was concluded that the CM is the best model in forecasting Sri Lankan stock returns.

In general, the risk of returns is measured by standard deviation or β coefficient of CAPM, but both methods are erroneous. In recent past, ARCH/GARCH models also were used for the purpose. The Engle's ARCH test confirmed the non existence of ARCH effect in Sri Lankan stock returns. The study suggested a new approach for measuring risk of returns. The theory of uniform circular motion (Newton's law) was applied in measuring risk. The suggested risk measurement, named as "Circular Indicator" was successful in measuring risk of returns.

Wave like patterns are common in the fields of Medicine, Agriculture, Meteorology and many others. It is recommended to test the CM on forecasting wave like patterns in these fields. Also it is worth testing the suitability of Circular Indicator, as a risk measurement in the above fields and many more.

Key Words: Fourier Transformation, Uniform Circular Motion, Covariance

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CHAPTER 1

INTRODUCTION

1.1 Background of the Research

Scientific forecasting plays a vital role in research and management of a large number of fields. Scientific forecasting is based on mathematical and statistical modeling. Mathematical models are deterministic. A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables. Deterministic models are not associated with any randomness; conversely statistical models are associated with the randomness. As such, statistical models have become more prominent in prediction, control and optimization.

1.2 Statistical Models

A statistical model is defined as a set of probability distributions on the sample space S (McCullagh, 2002). The three main categories of statistical models are: parametric statistical models, semi- parametric statistical models and non-parametric statistical models.

The Statistical models also can be broadly classified into two parts: univariate statistical models and multivariate statistical models. A univariate statistical model is an equation or set of equations explaining the behavior of a single random variable over time while the multivariate statistical models explain the joint behavior of two or more random variables. The univariate statistical modeling procedure is based on the past internal patterns in data to forecast the future and no external variables are required in forecasting. The basic concept of these methods is that the future values of a series are a function of past values. Univariate methods include: Moving Average Smoothing, Exponential Smoothing, Winters' method, Decomposition techniques, Box Jenkin’s ARIMA methods, Linear and Non-linear trend models (Stephen, 1998).

Multivariate models make projections of the future by modeling the relationship between a series and other series. It models future values of a series as a function of itself and values of other variables. Multivariate Regression and Vector Auto Regression (VAR) are some of the multivariate techniques.

1.3 Stock Market Forecasting

Risk and return are the most important concepts in financial markets (Pande, 2004). Investors expect higher returns at a lower risk; as such, they are very much concerned about the information on the risk and return of individual assets. Therefore, forecasting risk and return of assets were of immense interest over the past decades.

Statistical techniques and soft computing techniques are the two main strands of stock market forecasting. Statistical techniques comprise of Fundamental analysis and Technical analysis. The fundamental analysis involves analyzing the economic factors or characteristics of a company, namely; company value, company earnings, book-to-market equity etc. On the other hand, the interest in the technical analysis is the price movements and trading volume in the market.

Modern Portfolio theory of Markowitz (1952) was one milestone of fundamental analysis. Tobin (1958), Treynor (1961), Sharpe (1964), Lintner (1965), Mossin (1966), Black (1972) and many others contributed to the development of the idea of Markovitz (1952). Their combined output is known as the Capital Asset Pricing Model (CAPM), which is given by the formula;

$$E(R_i) = R_f + \beta[E(R_m) - R_f] \tag{1-1}$$

Where, $E(R_i)$ is the expected return of i^{th}company assets, $E(R_m)$ is the expected return of the market, R_f is the risk free rate of return and $\beta = \frac{Cov(R_{i,}R_m)}{\sigma_m^2}$. The β coefficient, which is known as the risk factor is the key parameter in CAPM. It is considered that, if $\beta = 0$, the share price is not at all correlated with the market, therefore no risk. If $\beta = +1$, an average level of risk. If $\beta > 1$, security returns fluctuates more than the market returns, at high risk. If $\beta < 1$, asset inversely follows the market (Pande, 2004). CAPM has been widely used in stock market forecasting all over the world.

In addition to CAPM, the Vector Auto Regression (VAR) models, exponential smoothing, Auto Regressive Integrated Moving Average (ARIMA) models, and Artificial Neural Network (ANN) are used in forecasting returns. In general, risk of a security is measured by the variance of returns or beta (β) factor of CAPM (Pande, 2004). In recent past, some scholars have attempted to measure the risk by Auto Regressive Conditional Heteroscedasticity (ARCH) and Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) models.

1.3.1 Share Trading in Sri Lanka

Share trading in Sri Lanka commenced in the 19th century, with the formation of Colombo Share Brokers Association. Later it was renamed as ‘Colombo Stock Exchange (CSE). According to official website of CSE, 295 companies are listed for year 2016, representing twenty business sectors. All Share Price Index (ASPI) and S&P SL20 are the main market indices. In addition, the market performance of each sector is indexed by corresponding sector indices.

Fundamental Analysis approach of asset pricing, CAPM has been used for the Sri Lankan share market forecasting. Beta (β) coefficients for listed companies are published by CSE on quarterly basis and the investors use them for investment decisions.

1.4 Research Problem

CAPM model has been subjected to extensive empirical testing in the past few decades. The central assertion of the CAPM is that, there exist a linear relationship between the expected return and the market risk (β). This was first argued by Banz (1981). Introducing the size effect for the explanation of returns, he showed that the average returns of stocks are negatively related to the market equity (ME). Black, Jensen and Scholes (1972); Fama and MacBeth (1973) found that the CAPM was valid for pre –

1969 period, but not afterwards. Bhandari (1988) has found that; risk (β), market equity and leverage together explain average returns. Stattman (1980); Rozenberg, Reid and Lanstein (1985); Chan, Hamao and Lakonishok (1991) have found that average returns are positively related to the book-to-market equity (BE/ME). Basu (1983) has shown that the earnings to price ratios (E/P) help explaining the cross section of average returns. Fama (1992) has shown that, only the firm size and BE / ME capture the cross sectional variation of average returns. Those findings have given considerable evidence that the risk itself cannot explain the returns of individual securities and hence the portfolio returns. Nimal (1997) and Samarakoon (1997) have confirmed the validity of the above findings for Sri Lankan stock market, But the Sri Lankan stock market still depends on the CAPM.

Literature revealed that Konarasinghe & Pathirawasam (2013); Rathnayaka, Seneviratna, & Nagahawatta (2014) and many others have attempted to establish a suitable forecasting technique for the Sri Lankan share market. But the forecasting ability of suggested methods was not satisfying, thus it is essential to develop suitable techniques for the Sri Lankan share market for forecasting returns.

Risk of a security is measured by the variance of returns or beta (β) factor of CAPM. If the observations of a data series are independent, then the variance is a suitable measure of dispersion. But time series data are generally auto correlated, as such, the variance may not be appropriate in measuring the risk of returns. Also the literature revealed that the β does not exist for many markets. Therefore, a clear knowledge gap exists in measuring risk of returns.

Research questions identified on this basis were:

i. What are the patterns of Sri Lankan share market returns?
ii. What would be the covariance structure of Sri Lankan share market returns?
iii. Is β coefficient suitable to measure the risk of returns of individual companies of the Sri Lankan share market?
iv. Is Auto Regressive Integrated Moving Average (ARIMA) model capable of forecasting Sri Lankan stock returns?
v. Does frequency domain approach; spectral analysis is successful in forecasting Sri Lankan stock returns?
vi. How to measure the stability (risk) of individual companies in share market performances?

1.5 Objectives of the Study

On view of the above, the objectives of this study were as follows;

Primary Objectives

i. To develop statistical models for forecasting Sri Lankan share market returns.
ii. To develop an indicator to compare the relative stability or risk of individual companies in share market performances.

Secondary Objectives

i. To identify the patterns of Sri Lankan stock returns.
ii. To identify the covariance structure of Sri Lankan share market and make forecasts based on covariance.
iii. To test the suitability of ARIMA models in forecasting Sri Lankan stock market returns.
iv. To test the applicability of Fourier transformation in forecasting Sri Lankan stock market returns.
v. To apply the Newton's law of uniform circular motion in measuring risk of returns.

1.6 Significance of the Study

Firms have two types of assets: real assets and financial assets. Real assets are physical assets; financial assets are shares and bonds. The firms issue securities to investors in the primary capital markets. The securities issued by firms are traded in the secondary capital markets (stock exchanges). Return of a single asset or a share is defined as sum of the dividend yield and the capital gain. The dividend yield totally depends on the dividend decision of a firm. As such, the major concern of the investors is the capital gain yield.

Share trading is important to the investors, industries as well as the entire economy of a country. Whenever a company wants to raise funds, it can issue shares of the company while an investor can get the part ownership of the company through buying shares. Also a country can gain foreign investment for the development via stock market. As such, a healthy stock market has been considered indispensable for the economic growth. Share trading contribution to the Sri Lankan Gross Domestic Product (GDP) has been increasing over the years. In year 2014, market capitalization of CSE was 2.4 trillion rupees, corresponds to approximately 1/3 of the GDP (Wikipedia, 2016).

Stock market investments are high risk investments. Thus, forecasting is the most important activity that helps to judge the market risk and grab scarce opportunities. Javad (1993) pointed out the importance of an efficient pricing mechanism to a stock market. According to the author, “a major factor hindering the foreign investment in a market is lack of information about the price or return behavior of the market”. CSE Annual Report (2014) revealed that domestic investor’s attraction towards share trading has increased while foreign investor’s attraction has decreased. This may be due to the inefficiency of the forecasting mechanism in the Sri Lankan share market.

Some studies have shown that the risk factor (β) is not important at all in stock market forecasting. As such it is essential to develop an indicator to measure the investment risk.

Objectives of the present study were twofold: to test existing methodologies in forecasting returns and to find new knowledge in forecasting. Outcomes of the study contribute not only to the Sri Lankan financial market but also to the other capital markets. Also new knowledge found in the study may applicable to the other fields as well.

1.7 Organization of the thesis

This study consists of five chapters. The first chapter carries an introduction and the second chapter reviews a substantial number of previous researches which gives insight into statistical modeling in financial markets. Chapter three carries methodology and theoretical background of the study. Results and data presentation are in chapter four. Chapter five contains the conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.1 Developments in Stock Market Forecasting

Scientific forecasting in share market returns has a long history going back to 1950's. With the increasing importance in forecasting share returns, a large number of studies focused on it from all over the world. Those studies can be broadly categorized into two parts: studies based on Statistical techniques and Soft Computing techniques (Ayodele**,** Aderemi & Charles, 2014). Statistical techniques are again subdivided into Fundamental analysis based studies and Technical analysis based studies (Konarasinghe & Pathirawasam, 2013). The fundamental analysis is involved in analyzing the economic factors or characteristics of a company; namely, company value, company earnings, book-to-market equity etc. The technical analysis is interested in the price movements and trading volume in the market. Artificial neural networks (ANNs) and Neurofuzzy are the soft computing techniques widely used in stock market forecasting. As this study was focused on technical analysis, the literature review was restricted to the technical analysis of financial market forecasting.

Forecasting stock returns by technical analysis goes back to findings of the Osborne (1959). Osborne's study was based on the Brownian motion which is also known as the particle theory. Osborne (1959) showed that the logarithms of common stock price changes also have a probability distribution similar to a particle in Brownian motion.

According to him, if $Y = \ln[P(t+\delta t)/P_0(t)]$ where *P (t+δt)* and *P₀ (t)* are the prices of the same random choice stock at random times *(t+δt)* and *t*, then the steady state distribution function of Y is,

$$\varphi(Y) = \exp\left[(-Y^2 / 2\sigma^2 \delta t) / (\sqrt{2\pi\sigma^2 \delta t})\right] \tag{2-1}$$

He also showed that the expected value of the share price of a common stock (*P*) increases with time at a rate of 3% to 5% per year and the variance of *P* increases with the increasing number of transactions. Osborne has tried to address the price- volume relationship, but was not successful.

Followed by Osborne (1959), a large number of studies were focused on modeling stock prices or returns based on Regression models, Auto Regression models, exponential smoothing, Auto Regressive Integrated Moving Average (ARIMA) models, ARCH / GARCH models and Artificial Neural Network (ANN). A limited number of studies were based on Spectral analysis.

In relevance to the objectives of the study, the literature review was restricted to:

i. Studies based on Covariance analysis.
ii. Studies based on Regression analysis.
iii. Studies based on Vector Auto Regression models.
iv. Studies Based on ARIMA models.
v. Studies based on ARCH /GARCH models

vi. Studies based on Spectral analysis.

2.2 Studies based on Covariance Analysis

Covariance plays a key role in financial economics, especially in portfolio management. In financial markets, covariance is a measure of the degree to which returns on two or more risky assets move in tandem. A positive covariance means that the asset returns move together. A negative covariance means that the returns move inversely. The study of Markowitz (1952) was the first study based on covariance analysis in forecasting returns and risk. According to the author, expected value is the measurement of return and variance is the measurement of risk. The expected value and the variance of returns of a single asset are given by formulae;

$$E(R_i) = \sum_{i=1}^{n} r_i P(r_i) \tag{2-2}$$

$$Var(R_i) = E(R_i^2) - [E(R_i)]^2 \tag{2-3}$$

Markowitz (1952) was the base of the development of CAPM.

Fama and James (1973); Bhandari (1988); Fama and Kenneth (1999); Samarakoon (1977); Nimal (1997) and many others have partially taken the idea of Markowitz (1952), but have incorporated characteristics of firms: size, liquidity, earnings to price ratio etc. in forecasting stock returns. It was difficult to find the studies purely based on the covariance structure of the share market.

2.3 Studies based on Regression Analysis.

The Regression analysis investigates and models the relationship between a response variable and one or more predictors. Regression models can be categorized as: simple regression models, multiple regression models and logistic regression models. These can be either linear models or non-linear models. Simple regression and multiple regression model the relationship between numerical variables while logistic regression models the relationship between categorical response variable and numerical or categorical predictors. The Ordinary Least Square method is used in parameter estimation of both simple and multiple regressions and maximum likelihood procedure is used in logistic regression. It is mandatory to test several assumptions about errors of regression. They are: independence of errors (errors are not serially correlated), normality of errors and homoscedasticity (constant variance) of errors. If any of these assumptions are violated, then the forecasts yielded by a regression model are inefficient or misleading.

Following price- volume relationships have been tested in several studies:

$$Y = \alpha + \beta_i X + \varepsilon \tag{2-4}$$

$$Y = \alpha + \sum \beta_i X_i + \varepsilon \tag{2-5}$$

X_i's are the predictor variables and ε is the random error.

Study of Crouch (1970) has tested Regression models on daily share price changes and trading volume data. His study was based on the New York stock exchange. Data collection period was from September 2006 to March 2007. Results of the Crouch (1970) have given evidence for a positive linear relationship between the absolute price change and the trading volume. But R^2 of the models were below 50% for daily data: therefore the author has used hourly share price data and trading volume to improve the model. This is clearly a disadvantage in his method, because the stock market forecasting is practically not useful on hourly basis. Further, his data collection period, which is seven months, was not sufficient. It is mandatory to test model assumptions: normality of residuals, serial auto correlation of residual and homoscedasticity of residuals, but the author has not reported the results of them. In addition, the author also has not done the model verification.

Clark (1973) has applied the subordinate stochastic process for speculative price changes. In the study he has tested the following linear and non- linear models;

$$\begin{aligned} \Delta P &= \alpha_1 + \beta_1 V + \varepsilon \\ \log(\Delta P)^2 &= \alpha_2 + \beta_2 V + \varepsilon \\ \log(\Delta P)^2 &= \alpha_3 + \beta_3 \log V + \varepsilon \\ Var(\Delta P) &= Ae^{\alpha v} \end{aligned} \tag{2-6}$$

Where, P is the share price change and V is the trading volume and ε is the random error term. His study has given evidence for the relationship between the share price change and the trading volume. During the time of Clark (1973), most of the academics and economists believed that share price changes and share returns are normally distributed. Clark (1973) found that the distribution of returns and the trading volumes follow Log-

Normal distribution. However, Clark (1973) has not performed tests on residuals and has not done the model verification. As such, the validity of the fitted models is doubtful.

Study of Timothy (1994) was based on the daily All Ordinaries Index (AOI) values and the trading volume statistics of the Australian stock market from April 1989 to December 1993. He has tested the linear and non linear regression models between the trading volume and the magnitude of returns;

$$V_t = \alpha_0 + \alpha_1 | R_t | + \alpha_2 D_t | R_t | + \varepsilon_t$$
$$V_t = \alpha_0 + \alpha_1 | R_t^2 | + \alpha_2 D_t | R_t^2 | + \varepsilon_t \qquad (2\text{-}7)$$

Where, R_t is the return on day t, V_t is the trading volume of day t, $D_t = 1$ if $R_t < 0$, and $D_t = 0$ if $R_t \geq 0$. His findings support the relationship between the price change and the trading volume, irrespective of the direction of the price change.

2.4 Studies Based on Vector Auto Regression (VAR) Models

Vector Auto Regression models were introduced by Sims (1980). These models can be used to characterize the joint behavior of variables. In VAR models each variable is a linear function of past lags of itself and past lags of the other variables. VAR models have been used in many fields including the financial management.

Timothy (1992) used VAR models on share returns and trading volumes. Weekly data of NASDAQ stock market (an American stock exchange.) from 1972 to 1986 has been used in the model testing. In the study, Timothy (1992) has tested the following univariate causal models;

$$R_t = \alpha_0 + \sum_{i=1}^{n} \beta_i R_{t-i} + \varepsilon_t$$

$$V_t = \lambda_0 + \sum_{j=1}^{n} \rho_j V_{t-j} + \varepsilon$$

$$V_t = \lambda_0 + \sum_{j=1}^{n} \rho_j R_t^2 + \gamma_j D_t R_t^2 + \varepsilon_t \tag{2-8}$$

And the following multivariate causal models;

$$R_t = \alpha_0 + \sum_{i=1}^{n} \beta_i R_{t-i} + \sum_{j=1}^{n} \gamma_j V_{t-j} + e_t \tag{2-9}$$

Where R_t is the return of week t, R_{t-i} is the return of i lag behind, V_t is the trading volume of week t, V_{t-j} is the trading volume of j lag behind and D_t is the dummy variable. He could not find any evidence for a multivariate causal relationship between returns and the trading volume, but he found evidence for univariate causality of returns. The author has tested regression coefficients, but has not tested modeling assumptions and not validated the selected model.

Saatcioglu and Starks (1998) have examined the stock price-volume relation in a set of Latin American emerging markets. They have collected data from; Argentina, Brazil, Chile, Colombia, Mexico, and Venezuela. Data collection period of the study was from January 1986 to April 1995. They have tested the VAR models;

$$V_t = \alpha_0 + \alpha_1 \ln\left(\frac{P_t}{P_{t-1}}\right) + \varepsilon_t$$

$$V_t = \alpha_0 + \alpha_1 \left|\ln\left(\frac{P_t}{P_{t-1}}\right)\right| + \varepsilon_t \tag{2-10}$$

Where (V_t) is the trading volume and (P_t) is the share price. They have found evidence for a return- volume relationship for four of the six markets, but not for all. They also have not performed tests for residuals and not done the model validation.

Chordia and Swaminathan (2000) have formed set of portfolios using data from USA stock databases. Data collection period was from 1963 to 1996. Daily and weekly equally weighted portfolio returns were modeled by;

$$R_{A,t} = \alpha_0 + \sum_{i=1}^{k} a_i R_{A,t-i} + \sum_{i=1}^{k} b_i R_{B,t-i} + \varepsilon_t$$

$$R_{B,t} = \alpha_0 + \sum_{i=1}^{k} c_i R_{B,t-i} + \sum_{i=1}^{k} d_i R_{A,t-i} + \varepsilon_t$$

$$R_{0,t} = \alpha_0 + \sum_{i=-k}^{k} \beta_{0,i_i} R_{m,t-i} + v_{0,t} \tag{2-11}$$

Where, $R_{A,t}$ is the return on the lowest trading volume portfolio of *A* on day *t*, $R_{B,t}$ is the return on the highest trading volume portfolio *B* on day *t*, $R_{0,t}$ is the return of zero net investment portfolio on day *t*. The authors have concluded that the tested models are suitable in forecasting. But R^2 of all the tested models were low. They also have not performed model verifications and test for errors. As such, their selected models may not be suitable for forecasting.

Study of Wen-Hsiu, Hsinan&Chwan-Yi (2004) is similar to the study of Chordia and Swaminathan (2000). Wen-Hsiuet. al. (2004), have used data from the Taiwan stock market from January 1991 to December 2002. Results of their study were different from the previous studies; they have not found a causal relationship between the returns and the trading volume.

Heimstra and Johathan (1994) have tested bidirectional causality between returns and the volume in New York Stock Exchange. Data collection period of the study was 1915 to 1990. They have tested linear auto regression models and non-linear auto regression models. The results of their study provide evidence for significant bidirectional nonlinear causality between returns and volume.

Guillrmo, Roni, Gideon and Jiang (2002) have studied the dynamic relation between return and volume of individual stocks listed on New York Stock Exchange and American Stock Exchange. They have used daily data from 1993 to 1998 and tested the auto regression model with interactions;

$$R_{i,t+1} = \alpha_i + \beta_i R_{i,t} + \lambda_i V_{i,t}.R_{i,t} + \varepsilon_{i,t+1} \qquad (2\text{-}12)$$

Where $R_{i,t}$ is return of i^{th} company on day t and $V_{i,t}$ is trading volume of i^{th} company on day t. Results of the study supported the return –volume relationship with interactions.

Gong-Meng, Michael and Oliver (2001) studied the dynamic relation between the stock returns, trading volume, and volatility. Their study was based on the daily market price index and the trading volume series from 1973 to 2000 for the nine largest stock exchanges: New York, Tokyo, London, Paris, Toronto, Milan, Zurich, Amsterdam, and Hong Kong. The models tested were;

$$R_t = \alpha_0 + \sum_{i=1}^{5} \beta_i R_{t-i} + \sum_{j=1}^{5} \gamma_j V_{t-j} + \varepsilon_t$$
$$V_t = \lambda_0 + \sum_{i=1}^{5} \rho_i V_{t-i} + \sum_{i=1}^{5} \varsigma_j R_{j-i} + \varepsilon_t \tag{2-13}$$

Where R_t is the return on day t, V_t is the trading volume of day t. According to the results of the study, bi-directional relationships exist only for some countries, but not for all. The authors have highlighted the importance of studying the joint behavior of stock prices and the trading volume.

Jianping, Olesya and Lubomir (2002) have examined the dynamic relation between the return and the volume of individual stocks in Russia and the other emerging markets. Their study was focused on the 28 large companies listed in the Russian Trading System (RTS). The daily closing prices and the daily trading volume from 1995 to 2001 were used to test the following models with interactions;

$$R_{i,t+1} = \alpha_i + \beta_i R_{i,t} + \lambda_i V_{i,t} . R_{i,t} + \varepsilon_{i,t+1}$$
$$R_{i,t+1} - R_{m,t+1} = \alpha_i + \beta_i (R_{i,t} - R_{m,t}) + \lambda_i (V_{i,t} - V_{m,t}) \ .(R_{i,t} - R_{m,t}) + \varepsilon_{i,t+1} \tag{2-14}$$

Where $R_{i,t}$ is the return of i^{th} company on day t, $V_{i,t}$ is the trading volume of i^{th} company on day t and $V_{m,t}$ is the trading volume of the market on day t. They have found strong evidence for the return volume- relationship.

Ciner (2003) has tested the causality models on small-capitalization firms in the US and France. Data used in the study were S&P 600 and the NM stock indices from 1995 to 2002. They have tested the VAR including a dummy variable D_i to account for the day of the week and month of the year effects in stock returns;

$$R_t = \alpha_0 + \sum_{i=1}^{l} \beta_i R_{t-i} + \sum_{j=1}^{l} \gamma_j V_{t-j} + \sum_{i=1}^{k} D_i + \varepsilon_t$$
$$V_t = \lambda_0 + \sum_{i=1}^{l} \rho_i V_{t-i} + \sum_{i=1}^{l} \varsigma_j R_{j-i} + \sum_{i=1}^{k} D_i + \varepsilon_t$$

(2-15)

Ciner (2003) also has confirmed the return- volume relationship for both US and France stock markets.

Xiangmei, Nicolaas&Yanrui (2003) have examined the relation between the trading volume and the stock returns for two Chinese A-share markets and ten individual companies of the energy sector. They have used share price indices, trading volume and share price data from 1997 to 2002 and have tested the linear regression models and auto regression models (2-13). They also have found strong evidence for the causal relationship between the returns and the trading volume.

Kamath (2007)'s study was based on the Nascent stock exchange of Turkey. The study has utilized the daily data of the Istanbul Stock Exchange from 2003 to 2006 to test the causality between daily index returns and daily volume, by using models (2-13). Findings of this study also supported the causality between the returns and trading volume.

Malabika, Srinivasan and Devanadhen (2008) have examined the relationship between the stock price changes and the trading volume for Asia-Pacific Stock Market. Data collection period was from year 2004 to 2008. The results of the study have evidenced for the significant relationship between trading volume and the absolute value of price changes for most of the selected companies.

Sarika and Balwinder (2009), have examined the return- volume relationship using daily data of the Sensitive Index (SENSEX).They have utilized the data from October 1996 to March 2006. The study has provided evidence for the causal relationship, given in equations (2-13).

Naliniprava (2011) has investigated the dynamic relationship between the stock return and the trading volume of the Indian stock Market and evidenced for bi-directional causality between trading volume and stock return.

Habib (2011) has investigated the joint dynamics of stock returns and trading volume (equations 2-13) for the data collected from Egyptian Securities Exchange (ESE). His analysis has not supported the existence of a causal relation trading volume and the stock returns.

Ong Sheue and Ho Chong (2011) have tested the VAR models, 2-13, using data from Malaysia and Singapore stock markets. They have found evidence for a significant bidirectional nonlinear causality between the returns and the trading volume in the Malaysian stock market.

Marwan (2012) has examined the causal relationship between returns and trading volume in the Palestine Exchange. Models 2-13, were tested on weekly trading volume and returns over the period from October 2000 to August 2010. The results supported the causality between the returns and trading volume.

Konarasinghe & Pathirawasam (2013) have tested the causal relationship between the returns and the trading volumes in the Sri Lankan share market, adopting models 2-13. Monthly total market returns and trading volumes, monthly sector returns and trading volumes from year 2005 to 2011 were used for model testing. The results of multivariate tests have revealed that there was no causal relationship between the market returns and the trading volumes. Further, they have found that the stock returns were auto-correlated and stationary while the trading volumes were auto-correlated but not stationary.

2.5 Studies Based on ARIMA Models

The ARIMA model was introduced by Box and Jenkins in 1970. In the ARIMA model, future value of a variable is a linear combination of past values and past errors, expressed as;

$$\phi_p(B)\Delta^d Y_t = \theta_q(B)\varepsilon_t \tag{2-16}$$

Where, Y_t is the actual value, ε_t is the random error at time t, φ_p and θ_q are the coefficients of autoregressive and moving average, respectively. B is the back shift operator.

Ayodele, Aderemi and Charles (2014-a) have tested the ARIMA models on forecasting the stock prices of the New York Stock Exchange (NYSE) and the Nigeria Stock Exchange (NSE). The data collection period of the study was from 1995 to 2011. The authors have considered: Bayesian Information Criterion (BIC), standard error of regression, adjusted R^2, LBQ-statistics and correlogram to select the best fitting models. The study has given strong evidence for the suitability of ARIMA in short term prediction of share prices. Their study was focused only on the developed markets, not on the emerging markets; as such, no evidence can be obtained for the suitability of ARIMA in forecasting emerging markets.

Prapanna, Labani and Saptarsi (2014) have done a study titled "Study of Effectiveness of Time Series Modeling (ARIMA) in Forecasting Stock Prices". The study was focused on Indian stock market. Eight companies from seven business sectors of National Stock Exchange were selected for the study. The data collection period was from April 2012 to February 2014. The Akaike Information Criteria (AIC) was used to quantify the goodness of fit of the model and the model with the least AIC was selected as the best fitting model. The Mean Absolute Error (MAE) was used to assess the forecasting ability of the selected models. Authors have concluded that the forecasting accuracy of models for all the companies were above 85%. But, the sampling technique was not mentioned in the study. Also residual tests were not performed on the selected models. As such their conclusion is quite doubtful.

Emenike (2014) has published an article titled “Forecasting Nigerian Stock Exchange Returns: Evidence from Autoregressive Integrated Moving Average (ARIMA) Model”. The author has used ARIMA models and forecasted stock prices of the Nigerian Stock Exchange (NSE). Monthly All-Share Indices of the NSE from January 1985 to December 2009 were used for the model fitting and validation. ARIMA (1, 1, 1) was selected as the best fitting model, but forecasted returns did not match with actual returns.

Many researchers have compared the forecasting ability of ARIMA with Neural Fuzzy and ANN and given considerable evidence for the success of ARIMA in the share market forecasting. Ayodele, Aderemi and Charles (2014-b) have compared ARIMA and Artificial Neural Networks models for stock price predictions. The study has utilized stock data of New York stock Exchange for the period from August 1988 to February 2011. The stationary of the series were tested with the help of a correlogram. Bayesian Information Criterion (BIC), standard error of regression, adjusted $R^{2,}$ Q-statistics, ACF and PACF for residuals were used in ARIMA model selection. Percentage Error (PE) was the measurement of errors. The study has concluded that both ANN and ARIMA models serve well in stock price prediction. Also, they have shown that the ARIMA models are more superior to ANN in forecasting.

Rosangela, Ivette, Lilian, and Rodrigo (2010) have compared ANN, Neural fuzzy and ARIMA for Brazilian stock index forecasting. They also have identified the forecasting ability of ARIMA in forecasting the stock returns.

2.6 Studies based on ARCH /GARCH models

Traditional time series models assume a constant one-period forecast variance. Engle (1982) generalize this implausible assumption, introducing a new class of stochastic processes called Auto-Regressive Conditional Heteroscedasticity (ARCH) processes. ARCH is a mean zero, serially uncorrelated processes with non constant variances conditional on the past, but constant unconditional variances. For such processes, the recent past gives information about the one-period forecast variance. The Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) model is an improvement of ARCH model, by Bollerselev (1986).

In other words, an ARCH (q) process is one for which the variance at time *t* is conditional on observations at the previous *m* times, and the relationship is;

$$Var(Y_t / Y_{t-1}) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_1 Y_{t-i}^2 + e_t \tag{2-17}$$

A GARCH (p,q) model uses values of the past squared observations and past variances to model the variance at time *t*;

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_1 Y_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + e_t \tag{2-18}$$

Where; Y_t is the observation at time *t*, σ_t is the variance at time *t*, e_t is the random error at time t.

ARCH / GARCH models are extensively used in financial time series. Hsieh (1989) modeled Heteroscedasticity in Daily Foreign-Exchange Rates. The author has estimated ARCH and GARCH models for five foreign currencies; Canadian dollar, Swiss franc, Deutsche mark, Japanese yen and British pound, using 10 years of daily data. It finds that the ARCH and GARCH models remove all heteroscedasticity in price changes in all five currencies. Goodness-of-fit diagnostics indicate that exponential GARCH with certain non normal distributions fits the Canadian dollar extremely well and the Swiss franc and the deutsche mark reasonably well. Only one non normal distribution fits the Japanese yen reasonably well, but none fit the British pound.

Timothy (1994) has tested GARCH models on the trading volume. The study was based on the daily All Ordinaries Index (AOI) values and the trading volume statistics of the Australian stock market from April 1989 to December 1993. Author has showed that the GARCH model is suitable for volatility explanations.

Bekaert and Harvey (1997) has applied GARCH models to twenty emerging markets. The authors have used US dollar returns of those markets for the period, January 1976 to December 1992. They have compared the conditional and unconditional volatility measures and confirmed the superiority GARCH (1,1) model in explaining volatility of the markets.

Olowe (2009) investigated the relation between stock returns and volatility in Nigeria using E-GARCH model. Using daily returns over the period for January 2004 to January 2009, volatility persistence, asymmetric properties and risk return relationship were investigated. The study found little evidence on the relationship between stock returns and risk as measured by its own volatility. The study found positive but insignificant relationship between stock return and risk. The result shows the banking reform in July 2004 and stock market crash since April 2008 negatively impacts on stock return while insurance reform and the global financial crisis have no impact on stock return. The stock market crash of 2008 is found to have contributed to the high volatility persistence in the Nigerian stock market especially during the global financial crisis period. The stock market crash is also found to have accounted for the sudden change in variance.

Peiris and Peiris (2011) examined the volatility of different sectors in Colombo Stock Exchange (CSE) and how the macro economic factors affect on the volatility by fitting ARCH and the GARCH using monthly time series data of 20 sectors in CSE for the period 2005 to 2010. Authors have used ARMA model as mean equation, and the residuals of the fitted ARMA models were tested on ARCH/ GARCH models. Results found that sixteen out of twenty sectors in CSE has a significance volatile ($p<0.05$) and both ARCH and GARCH terms on the fitted models for individual sectors were significant ($p<0.05$).

Samayawardena, Dharmarathne and Tilakaratne (2015) have tested the volatility models for the stock indices of Colombo stock exchange and stock markets of United States, India and United Kingdom are highly useful to the investors. To capture the characteristics of volatility in the stock price series, GARCH models have been used. All Share Price Index of Colombo stock exchanges (ASPI), S & P 500 index of New York stock exchange, FTSE 100 of the London stock exchange and BSE SENSEX index of Bombe stock exchange have been considered in the study, and the study period is from 1st January 2004 to 1st January 2014. GARCH (1, 1) model was identified as the best model for the ASPI return series, EGARCH (1,1) model was identified as the best model for both the FTSE 100 and BSESENSEX indices return series and while S & P 500 return series is best expressed by EGARCH (2,1) model. The model adequacy of the selected models have been tested using the ARCH LM test, Correlogram of squared returns and Correlogram of standardized residuals, while Q-Q plot was applied to check the error distribution.

2.7 Studies based on Spectral Analysis.

Cyclical patterns are a common feature in natural sciences. The analysis of electrical signals; image processing; sound spectrograms are examples for that. There are two ways of viewing any type of a wave: in the time domain, or in the frequency domain. The traditional way of observing such waves is to view them in the time domain. The time domain is a record of what happens to a parameter of the system versus time or space. The time domain analysis is known as "Time Series Analysis". Waves generally have a period. A period is the distance between two peaks or troughs or time between two peaks or troughs of a wave. A closely related property of the wave period is the frequency. The frequency domain analyses a signal with respect to the frequency. This frequency domain representation of signals is called the "spectrum" of the signal and frequency domain analysis is known as the "Spectral Analysis". It is also known as the Fourier analysis.

Spectral analysis was initially established in natural sciences such as Physics, Engineering, Geophysics, Oceanography, Atmospheric science, Astronomy etc. and was not much used in the field of Economics. By 1959, John Von Neumann of Princeton University, UK has realized the applicability of Spectral analysis in economic time series. Neumann's idea was taken by his co-author Oskar Morgenstern and two other fellows: C. W.J. Granger and Michio Hatanaka. As a result Granger and Hatanaka could publish the book, "Spectral Analysis of Economic Time Series". Granger & Morgenstern (1963) was the first recorded application of Spectral analysis in financial markets. Their study was focused on New York stock market. They have used weekly share price data, weekly turnover and monthly share price data, for the period from 1939 to 1961, analyzed the price series by Spectral Analysis. Granger & Morgenstern (1963) have tested the periodic function;

$$R_t = V_t + a \cos \omega t \tag{2-19}$$

Where R_t is the return on period t and V_t is the trading volume on period t. But the results of the study were not successful as expected. Granger & Hatanaka (1964) have conducted a study using monthly share prices of the New York stock exchange for the period from January 1946 to December 1960. The results of their study were also not up to the level of satisfaction.

It was noted that fewer studies had been conducted on stock returns using Spectral analysis. As Granger and Hatanaka (1964) emphasized, it may be due to the lack of understanding in advanced mathematical techniques: Trigonometry, Calculus and Complex numbers.

According to the literature, CAPM and VAR were successfully applied in forecasting stock returns in large number of stock markets. But most of them were developed markets and few were emerging markets. The ARIMA also gained importance in forecasting share prices or returns in the recent past. Notably fewer applications were in Spectral analysis.

CHAPTER 3

MATERIALS & METHODS

3.1 Background to the Data

Pattern recognition of stock returns paves the path for model developments. It gives an insight about the trends, seasonal variations, cyclical variations and volatility of the time series. Therefore, the study was begun with time series pattern recognition of stock market returns. Stock market is a population consists of a number of subsets (sectors). For example, Colombo Stock Exchange (CSE) has 20 business sectors. These subsets are defined in a way that they are mutually exclusive and the elements of these subsets (listed companies of these sectors) are homogeneous in the nature of business. Therefore, returns of individual companies within the sector could move together or returns of individual companies could move with the total market. As such, it was intended to understand the covariance structure of the Sri Lankan share market and make use of the findings for forecasting individual company returns.

Konarasinghe & Pathirawasam (2013) has shown that the sector returns and total market returns of the Sri Lankan stock market are of stationary type. It may be true for the individual company returns as well. If so, the stock returns have a wave like pattern. A wave can be viewed either in a time domain or a frequency domain. Therefore the study was directed to the time domain analysis and the frequency domain analysis. Auto Regressive Integrated Moving Average (ARIMA) models were tested in the time domain analysis. Fourier transformation was applied on the returns in the frequency domain analysis.

In general, risk of a security is measured by the variance of returns or beta factor of CAPM (Pande, 2005). But both methods are erroneous. Hence the study was focused to develop an indicator to compare the relative risk of individual companies in their market performances. In recent past the ARCH /GARCH models have extensively used to explain the risk of returns. As such, the existence of ARCH effect was tested on the individual company returns of Sri Lankan share market. Then a totally new approach was tested on measuring the stability of individual companies in market performances. That is, the theory of Uniform Circular Motion was used to explain the risk of returns.

3.2 Population and Sample of the Study

Listed companies of Colombo Stock Exchange (CSE) in year 2014 were the population of the study. The population consisted 20 business sectors: Plantation (PLT), Oil palms (OIL), Land & Property (L&P), Motors (MTR), Manufacturing (MFG), Telecommunication (TLE), Stores supplies (S&S), Trading (TRD), Services (SRV), Power & energy (P&E), Investment trust (INV), Hotels & Travels (H&T), Heath care (HLT), Footwear & Textile (F&T), Information Technology (IT), Diversified Holdings (DIV), Construction engineering (C&E), Chemicals and Pharmaceuticals (C&P), Beverage Food and Tobacco (BFT) and Bank, Finance and Insurance (BFI). There were 294 companies listed in year 2014. Daily closing share prices of individual companies, monthly indices of business sectors and All Share Price Indices (ASPI) from year 1991 to year 2014 were obtained from CSE.

The population of the study consist twenty business sectors; as such it was intended to use stratified sampling technique with proportions. But, some of the business sectors have only few companies, for examples, the sector IT has only one company, the sector C & E has only three companies etc. Therefore the simple random sampling technique was adopted; a sample of fifty companies was selected for the data analysis, excluding the companies not having continuous trading.

Monthly average share returns for individual companies were calculated by a standard formula,

$$R_t = \left(\frac{P_t - P_{t-1}}{P_{t-1}} \right) 100 \tag{3-1}$$

Where; P_t is the share price at time t. Monthly returns of business sectors were calculated by formula;

$$R_S = \left(\frac{I_t - I_{t-1}}{I_{t-1}} \right) 100 \tag{3-2}$$

Where I_t is the sector index of month t. Monthly total market returns were calculated by formula;

$$R_M = \left(\frac{ASPI_t - ASPI_{t-1}}{ASPI_{t-1}} \right) 100 \qquad (3\text{-}3)$$

Where, *ASPI* is the All Share Price Index.

Outlier Adjustment

Outliers are extremely large or small values outside the overall pattern of a data set. The outlier detection and adjustment are essential in data analysis. Boundaries of outliers are defined in many ways. Following rule is often used in outlier detection (Attwood, Clegg, Dyer and Dyer, 2008).

$$\begin{aligned} L &= Q_1 - 1.5 * IQR \\ U &= Q_3 + 1.5 * IQR \end{aligned} \qquad (3\text{-}4)$$

Where Q_1, Q_3 are the lower quartile and upper quartile respectively, *IQR* is the inter quartile range, *L* is the lower boundary and *U* is the upper boundary. Any data value above *U* or below *L* was considered as outliers. Such data points were adjusted by taking moving average of order three, using a computer program written in MATLAB (Appendix 1.1).

Accuracy of the program is based on two assumptions; first three values of the array are not being outliers, three consecutive outliers have not occurred. Outliers were manually adjusted, when one or two of the assumptions were violated.

3.3 Statistical Methods and Terminology Used in the Study

Following terminologies and statistical techniques were extensively used in the study.

Covariance and Correlation

Covariance and correlation measure a certain kind of dependence between two or more random variables. Suppose that X and Y are real-valued, jointly distributed random variables with means E(*X*), E(*Y*) and variances Var(*X*), Var(*Y*), respectively; then the covariance between X and Y is defined as;

$$Cov(XY) = E[X - E(X)][Y - E(Y)] \qquad (3\text{-}5)$$

For a population of size *N*, the covariance can be written as;

$$Cov(X,Y) = \frac{\sum_{i=1}^{N}(x_i - \overline{x})(y_i - \overline{y})}{N} \qquad (3\text{-}6)$$

And for samples of size *n*, it is given as;

$$Cov(X,Y) = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{n-1} \qquad (3\text{-}7)$$

In the special case when *X*=*Y*,

$$Cov(X,X)=\frac{\sum_{i=1}^{N}(x_i-\bar{x})^2}{N}=\sigma_x^2 \tag{3-8}$$

Correlation is a scaled version of covariance. The correlation between X and Y is defined as;

$$Cor(X,Y)=\frac{Cov(X,Y)}{\sqrt{Var(X).Var(Y)}} \tag{3-9}$$

The sign of the covariance or correlation shows the tendency in a linear relationship between two variables. Both the covariance and correlation indicate whether the variables are directly or inversely (positively or negatively) related. The correlation also tells the degree to which the variables tend to move together.

Auto Correlation

Auto correlation is the correlation between observations of a time series separated by k time units or k lags. The population ACF of lag k is defined as;

$$\rho_k=\frac{\gamma_k}{\gamma_0} \tag{3-10}$$

Where γ_k is the covariance at lag k and γ_0 is the variance. The sample autocorrelation covariance at lag k is given by;

$$\hat{\rho}_k = \frac{\sum_{t=1+k}^{n}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2} \tag{3-11}$$

The plot of autocorrelations is called the correlogram. It can be used for pattern recognition and as a method for testing stationary of a series (Stephen, 1998).

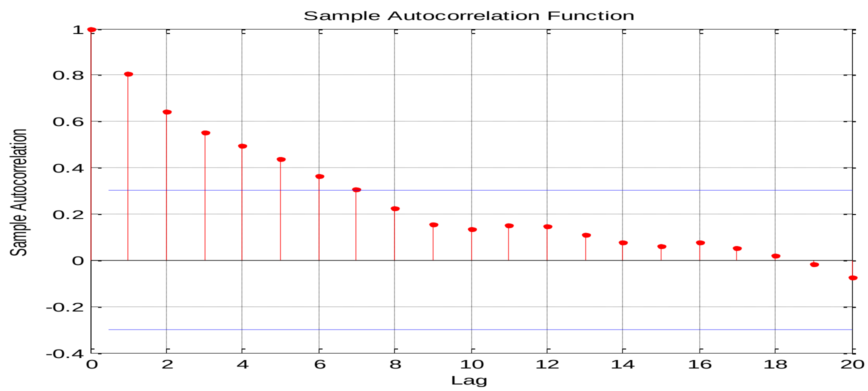


Figure 3.1: ACF of a Series with a Trend Component

The decreasing pattern of Figure 3.1 suggests a trend component in the series.

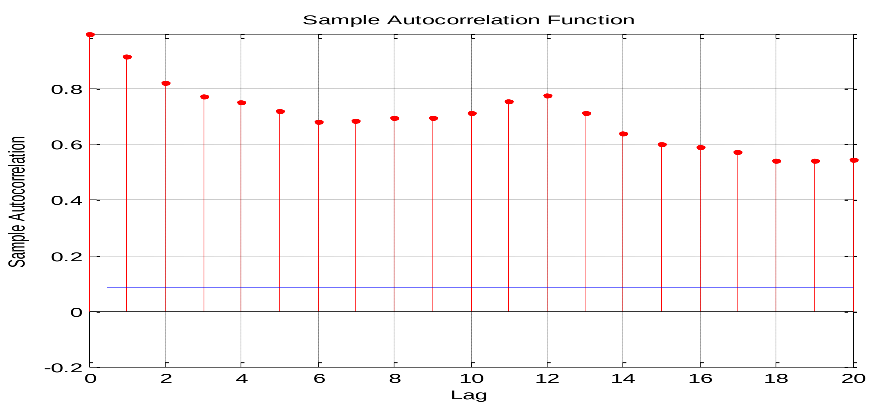


Figure 3.2: ACF of a Series with a Seasonal Pattern

Figure 3.2 shows a repeating pattern in ACF. It suggests a presence of seasonal component in the time series.

Stationary Stochastic Process

A stochastic process with a discrete time parameter is said to be stationary (or stationary in the strict sense) if the distribution of $y_{t_1}, y_{t_2}, ..., y_{t_n}$ is the same as the distribution of $y_{t_1+t}, y_{t_2+t}, ..., y_{t_n+t}$ for every finite set of integers $\{t_1, t_2, ..., t_n\}$ and for every integer t (Anderson, 1971). A stochastic process is said to be stationary, if its mean and variance are constants over time and the value of the covariance between the two periods depends only on the distance (gap or lag) between the two time periods and not the actual time at which the covariance is computed. In the time series literature, such a stochastic process is known as a weakly stationary or covariance stationary process (Gujarati, Porter & Gunasekar 2009). A special type of stochastic process is purely random or white noise, process. A white noise process has a zero mean and a constant variance. A White noise process is serially uncorrelated. The Random Walk Model (RWM), given by formula (3-12) is a white noise process.

$$Y_t = Y_{t-1} + \varepsilon_t \qquad (3\text{-}12)$$

The Auto Correlation Function (ACF) and Unit Root Tests are used to test the stationary of a time series. The ACF of a white noise is shown in the Figure 3-3;

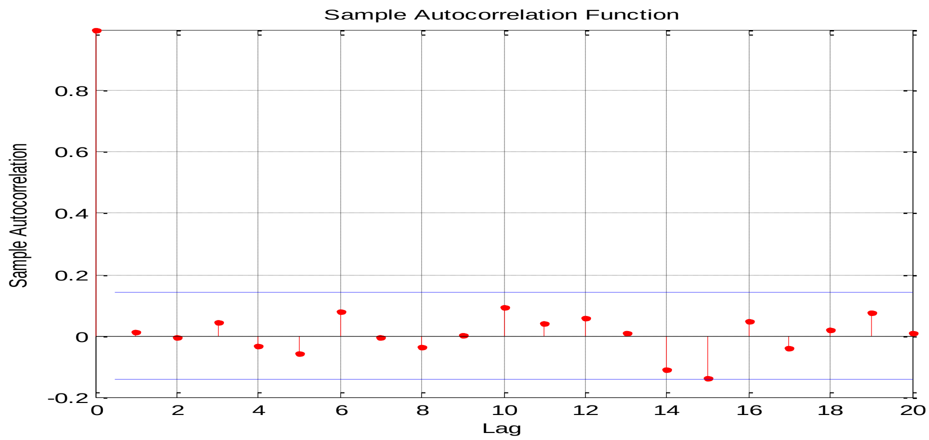


Figure 3.3: ACF of a White Noise

A White noise process is a stationary process, but has no significant lags in ACF as there are no serial correlations in the series.

Unit Root Test

Markov first order autoregressive model is given by equation (3-13);

$$Y_t = \rho Y_{t-1} + \varepsilon_t \tag{3-13}$$

The hypothesis test for ρ is known as the unit root test, where;

H_0: ρ=1, H_1: ρ <1

If the null hypothesis is rejected, series $\{Y_t\}$ is said to be stationary, if not $\{Y_t\}$ resembles a random walk model. However, the above hypothesis test cannot be performed by a t-test, because the t-test is biased in the case of a unit root (Gujarati et.al. 2009). Adjusting equation (3-13);

$$\begin{aligned} Y_t - Y_{t-1} &= \rho Y_{t-1} - Y_{t-1} + \varepsilon_t \\ \Delta Y_t &= (\rho - 1) Y_{t-1} + \varepsilon_t \\ \Delta Y_t &= \delta Y_{t-1} + \varepsilon_t \end{aligned} \tag{3-14}$$

The Augmented Dickey-Fuller Test (ADFT) is one of the tests used for unit root, as follows;

H_0: δ=0, H_1: $\delta < 0$

If the null hypothesis is not rejected, then $\rho < 1$; the series has a unit root. Then it concludes that the series is not covariance stationary.

The MATLAB syntax for ADFT is;

H_0: The unit root exists, H_1: The unit root does not exist

[h, pValue, stat, cValue, reg]= adftest(Y)

The result, h = 1; indicates that this test rejects the null hypothesis of a unit root against the autoregressive alternative. Then it concludes that the series is covariance stationary.

General Linear Processes (GLP)

A General Linear Process is a stationary stochastic process $\{Y_t\}$ which can be represented as a weighted linear combination of the present and past terms of a white noise. These include Auto Regressive (AR), Moving Average (MA), Auto Regressive Moving Average (ARMA) and Auto Regressive Integrated Moving Average (ARIMA).

AR_p Model

The Auto Regressive Process $\{Y_t\}$ of order p has the model;

$$Y_t = c + \sum_{i=1}^{p} \Phi_i Y_{t-i} + \varepsilon_t \tag{3-15}$$

Where Y_{t-i} are past observations of random variable Y_t.

MA_q Model

The Moving Average Process $\{Yt\}$ of order q has the model;

$$Y_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{3-16}$$

Where ε_{t-i} are past errors of random variable Y_t.

ARMA p,q Model

A model containing both AR and MA parts is known as a mixed model or the Auto Regressive Moving Average (ARMA) model. ARMA (p,q) model is:

$$Y_t = c + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \sum_{i=1}^{p} \varphi_i Y_{t-i} \tag{3-17}$$

ARIMA p,d,q Model

ARMA model is not valid if the series is not stationary; therefore, the differences are taken in order to achieve the stationary. Order of difference is given by d. The model is called the Auto Regressive Integrated Moving Average model which is given by the equation (2-17): $\phi_p(B)\Delta^d Y_t = \theta_q(B)\varepsilon_t$

Model Validation Methods

Goodness of fit tests and measurements of errors were used in the model validation of the study. The goodness of fit of a statistical model describes how well it fits a set of observations. The plots of residuals versus fits, Auto Correlation Functions (ACF) and Partial Autocorrelation Functions (PACF) of residuals and Ljung-Box Q statistics (LBQ) were used to test the independence of residuals. Histogram of residuals, Normal probability plot of residuals and Anderson Darling test were used to test the normality of residuals.

Measurements of Forecasting Errors

Forecasting is a part of a larger process of planning, controlling and/ or optimization. Forecast is a point estimate, interval estimate or a probability estimate. One of the fundamental assumptions of statistical forecasting methods is that an actual value consists of a forecast plus an error; In other words, "Error = Actual value – Forecast". This error component is known as the residual. A good forecasting model should have a minimum average of absolute error and zero average of error mean because it should over forecast and under forecast approximately the same (Stephen, 1998).

Measuring errors is vital in the forecasting process. The measurements of errors are divided into two parts; the Absolute measures of errors and the Relative measures of errors. Some absolute measures of errors are; Mean Error (ME), Mean Absolute Deviation (MAD), Sum of Squared Errors (SSE), Root Mean Squared Error (RMSE) and Residual Standard Error (RSE). The absolute measures of errors are very much dependent on the scale of measurement of the dependent variable. Also these measures do not allow comparisons of results over time or between time series. The relative measures of errors are capable in avoiding these disadvantages.

Some relative measures of errors are: Percentage Error (PE), Mean Percentage Error (MPE) and Mean Absolute Percentage Error (MAPE). However, relative measures of errors mislead when data values are extremely small (Stephen, 1998). Also relative measures become undefined when data values are equal to zero. As such absolute measures of errors are better to be used in such a situation. Stock returns, defined by formulae (3-1), (3-2) and (3-3) contain zero values. Therefore, relative measures of errors were not used in the study.

3.4 Pattern Recognition of Stock Returns

Pattern recognition helps to spot the suitable forecasting techniques. In this study, Box-plots, Time series plots and Auto Correlation Functions (ACF) were used for pattern recognition.

3.5 Identification of Covariance Structure of Sri Lankan Share Market Returns

Firstly, the covariance structure of sector returns and the total market were studied. Then the covariance between individual company returns and corresponding sector returns, the covariance between individual company returns and the total market returns were studied. Hence the individual company returns were modeled on the corresponding sector returns.

3.6 ARIMA Models on Forecasting Stock Returns

Covariance stationary of a series $\{Y_t\}$ is mandatory in ARIMA model testing. Differenced series are obtained and check for stationary when the original series does not meet the criteria. Differenced series are defined as follows;

First differenced series: $Y_t^{'} = Y_t - Y_{t-1} = (1-B)Y_t$ (3-18)

Second differenced series:

$$Y_t^{''} = Y_t^{'} - Y_{t-1}^{'} = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2}) = Y_t - 2Y_{t-1} - Y_{t-2} = (1-B)^2 Y_t \quad (3\text{-}19)$$

Where, B is the Back Shift operator and $BY_t = Y_{t-1}$. In general ARIMA procedure does not go beyond the third difference. The Auto Correlation Function (ACF), Partial Auto Correlation Function (PACF) and unit root tests are used to test the stationary of a series. When the stationary is confirmed, ACF and PACF are used to spot the suitable ARIMA models for testing. For example: if the first difference series is stationary, there exist a single significant spike at lag 1 in ACF and exponential decline in PACF, then ARIMA (0, 1, 1) would be a suitable model. Table 3-1 gives some rules of thumbs used in ARIMA model testing (Stephen, 1998).

ARIMA models were tested on the random sample of fifty companies.

Table 3.1: Behavior of ACFs and PACFs in ARIMA Process

Process	**ACFs**	**PACFs**
ARIMA (0,0,0)	No significant lags	No significant lags
ARIMA (0,1,0)	Linear decline at lag 1, with many	Single significant spike at lag 1
ARIMA (1,0,0)	Exponential decline, with first	Single significant spike at lag 1
ARIMA (1,0,0)	Alternating exponential decline,	Single significant negative spike
ARIMA (0,0,1)	Single significant negative spike	Exponential decline, with first
ARIMA (0,0,1)	Single significant positive spike	Alternating exponential decline

Auto Correlation Functions (ACF) and Partial Autocorrelation Functions (PACF) were obtained to test the stationary of the series and to test whether the series follow trend or seasonal patterns. Augmented Dickey Fuller test was used to confirm the stationary of the series. When the stationary was confirmed, several ARIMA models were tested on each series and the best fitting model was selected by comparing MSE, MAD and the results of goodness of fit tests.

3.7 Fourier Transformation on Forecasting Returns

Fourier transformation (FT) can be used to transform a real valued function f(x) into series of trigonometric functions (Philippe, 2008). FT has two versions; discrete transformation and continuous transformation. The discrete version of Fourier transformation is; $f(x) = \sum_{-\infty}^{\infty} a_n e^{-k\theta}$ (3-20)

According to De Moivre's theorem; $e^{-k\theta} = \cos k\theta + i \sin k\theta$ (3-21)

Where, *i* is a complex number. Therefore *f(x)* can be written as:

$$f_x = \sum_{k=1}^{n} a_k \cos k\theta + b_k \sin k\theta \quad (3\text{-}22)$$

Whare a_k and b_k are amplitudes, k is the harmonic of oscillation. The highest harmonic (k) is defined as (Stephen, 1998);

$$k = \begin{cases} n/2: & n \text{ even} \\ (n-1)/2: & n \text{ odd} \end{cases}$$

The Fourier transformation is incorporated to a uniform circular motion of a particle in a horizontal circle and basic trigonometric ratios.

A particle *P*, which is moving in a horizontal circle of centre O and radius *a* is given in Figure 3.4. The ω is the angular speed of the particle;

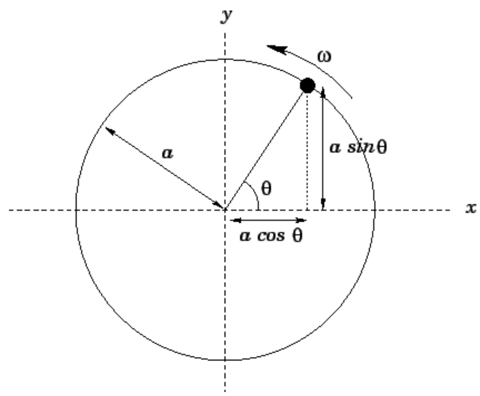


Figure 3.4: Motion of a particle in a horizontal circle

Angular speed is defined as the rate of change of the angle with respect to time. Then;

$$\omega = \frac{d\theta}{dt}$$

$$\int_0^{\theta} d\theta = \int_0^{t} \omega \, \mathrm{dt}$$

Hence, $\theta = \omega t$ (3-23)

Substitute (3-23) in (3-22); $f_x = \sum_{k=1}^{n} a_k \cos k\omega t + b_k \sin k\omega t$ (3-24)

At one complete circle θ=2π radians. Therefore, the time taken for one complete circle (T) is given by: $T = 2\pi / \omega$ (3-25)

Figure 3.5 and Figure 3.6 clearly show how to incorporate a particle in horizontal circular motion to trigonometric functions;

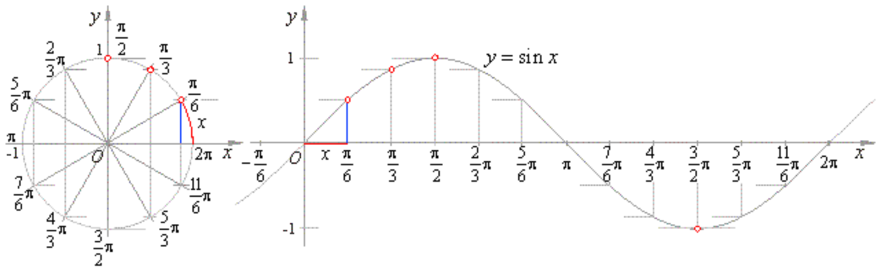


Figure 3.5: sine function and reference circle

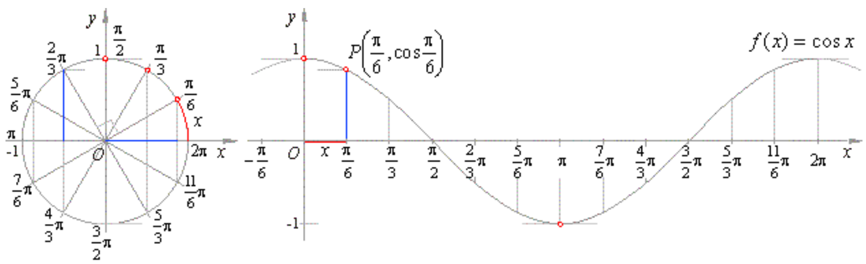


Figure 3.6: cosine function and reference circle

Reference to Figure (3-4); $\overrightarrow{op} = a(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$, where, a is the amplitude or wave height. A wave with constant amplitude is defined as a regular wave and a wave with variable amplitude is known as an irregular wave.

The concept of Fourier transformation is applied in the present study for explaining returns. In circular motion, the time taken for one complete circle is known as the period of oscillation. In other words, the period of oscillation is equal to the time between two peaks or troughs of sine or cosine function. If a time series follows a wave with f peaks in N observations, its period of oscillation can be given as;

$$T = \frac{total \text{ number of periods}}{\text{total number of peaks}} = \frac{N}{f} \tag{3-26}$$

Equating (3-25) and (3-26); $\frac{2\pi}{\omega} = \frac{N}{f}$

Then, $\omega = 2\pi \frac{f}{N}$ (3-27)

Hence the return at time t (R_t) was modeled as;

$$R_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \tag{3-28}$$

The model (3-28) was named as “Circular Model”.

The MATLAB program given in Appendix 1.2 was used for peak identification and ω calculation. The linear independence of trigonometric series; sin($k\omega t$) and cos($k\omega t$) were confirmed by correlation analysis. The Multiple regression technique was adopted for estimation of amplitudes a_k and b_k. The Circular model was tested on the sample of 50 companies.

3.8 Test the Existence of ARCH Effect on Returns

An uncorrelated time series can still be serially dependent due to a dynamic conditional variance process. A time series exhibiting conditional heteroscedasticity or autocorrelation in the squared series is said to have ARCH effects. Engle's ARCH test is a Lagrange multiplier test to assess the significance of ARCH effects.

Consider a time series; $Y_t = \mu_t + \varepsilon_t$ (3-29)

Where μ_t is the conditional mean of the process, and ε_t is an innovation process with mean zero. Suppose the innovations are generated as, $\varepsilon_t = \sigma_t Z_t$ (3-30)

Where, z_t is an independent and identically distributed process with mean 0 and variance 1. Thus, $E(\varepsilon_t \varepsilon_{t+h}) = 0$, for all lags h≠0 and the innovations are uncorrelated.

Define the residual series $e_t = Y_t - \hat{\mu}_t$ (3-31)

If all autocorrelation in the original series, Y_t, is accounted for in the conditional mean model, then the residuals are uncorrelated with mean zero. However, the residuals can still be serially dependent.

The Engle's ARCH test is such that;

$$H_0 = \alpha_0 = \alpha_1 = ... = \alpha_m = 0$$
$$H_1 = e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + ... + \alpha_m e_{t-m}^2 + u_t$$

The test is based on the *F* statistic. The MATLAB syntax for ARCH test is such that;

```
[h,p,fStat,crit] = archtest(e,'Lags',m)
```

The result *h* = 1 indicates the rejection of null hypothesis of no conditional heteroscedasticity and conclude that there are significant ARCH effects in the series.

3.9 Development of Stability Indicator for Returns

The development of a stability indicator was based on the uniform circular motion of a particle in a horizontal circle (Newton's law).

Theory of Uniform Circular Motion of a Particle in a Horizontal Circle

Reference to Figure (3.4), position vector of a particle at time *t* is;

$$\overrightarrow{op} = a\cos\theta\,\mathbf{i} + a\sin\theta\,\mathbf{j} = a(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

Radius of the circle (*a*) is a constant; therefore the position vector of the particle at time is a function of θ. But θ vary with time. As such, the magnitude of the velocity or speed of the particle can be obtained by differentiating the position vector of the particle with respect to t and the acceleration of the particle can be obtained by differentiating the velocity vector, with respect to *t*;

$$v = \left|\frac{d}{dt}\overrightarrow{op}\right|$$
$$= \left|\frac{d}{dt}a(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})\right|$$
$$= \left|a(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})\frac{d\theta}{dt}\right|$$
$$= a\,|-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}|\,.\left|\frac{d\theta}{dt}\right|$$
$$= a.1.\left|\frac{d\theta}{dt}\right|$$

Hence; $v = a\omega$ (3-32)

The acceleration of the particle is obtained by differentiating the velocity. That is

$$Accelerati\,on(\mathbf{a}) = \frac{d}{dt}\mathbf{v} = \frac{d}{dt}a\omega = a\frac{d^2}{dt^2}\theta$$

Hence; $$\mathbf{a} = a\omega^2 = \frac{v^2}{a} \qquad (3\text{-}33)$$

When the particle moves in a circle, it is constantly changing its direction. At all instances, the particle is moving tangent to the circle. Since the direction of the velocity vector is the same as the direction of the motion, the velocity vector is directed tangent to the circle; as such, the acceleration of the particle also tangent to the circle. Even though the particle is moving under the acceleration with a changing direction, it does not leave the circular path. Therefore, there should be force acting towards the centre of the circle which prevents particle leaving its locus. This force is named as the centripetal force (Hooker, Jennings, Littlewood, Moran and Pateman, 2009).

Using the Newton's second law of motion; $\mathbf{F} = m\mathbf{a}$ towards the centre;

$$\mathbf{F} = ma\omega^2 = m\frac{v^2}{a} \qquad (3\text{-}34)$$

The centripetal force (**F**) is directly proportional to the mass and the square of the velocity, but inversely proportional to the radius of the circle. In other words the stability of a motion of a particle depends on the mass of the particle, its velocity and the radius of the circular motion.

It can be shown that the radius of the reference circle in Figure (3.5) or Figure (3.6) is equal to the amplitude or height of the wave.

Proof: Radius of the reference circle = Amplitude of the wave

Reference to Figure (1), position of the particle at time t is;

P= (x, y) = (a $\cos\theta$, $a \sin\theta$)

Consider $y=a \sin\theta$ function;

The magnitude of the maximum or minimum value of a sine function is the amplitude of the wave. The maxima of sine functions occur at $\theta=2n\pi \pm (\pi/2)$ for n=0,1,2,3, and the minima occur at $\theta=2n\pi \pm (3\pi/2)$ for n=0,1,2,3,…

When θ=π/2, maximum value of y; $y_{max} = a.\sin(\pi/2) = a$

When θ=3π/2, minimum value of y; $y_{min} = a.\sin(3\pi/2) = -a$

Therefore, $|y_{max}|=|y_{min}|= a$ = amplitude

Consider $y=a \cos\theta$ function;

The magnitude of the maximum or minimum value of a cosine function is the amplitude of the wave. The maxima of cosine functions occur at $\theta=2n\pi$ for n=0,1,2,3, and the minima occur at $\theta=2n\pi \pm (\pi)$ for n=0,1,2,3,…

When θ=0, maximum value of y; $y_{max} = a.\cos(\pi/2) = a$

When θ=π, minimum value of y; $y_{min} = a = .cos(\pi) = -a$

Therefore, $|y_{max}| = |y_{min}| = a$ = amplitude.

The study shows that the share returns of a company follow a uniform circular motion. If mass of the particle (per share return) assumed to be 1, then;

$$F_{i,t} = r_{i,t} .\omega_{i,t}^2 \tag{3-35}$$

Where $F_{i,t}$ is the force making returns to be in a circular motion of a company *i* at time *t*, $r_{i,t}$ is the radius of the circular motion of *i*[th] particle at time *t* and $\omega_{i,t}$ is the angular speed of the circular motion at time *t*. As such, $F_{i,t}$ can be taken as the stability indicator of the market performances. That is; larger the $F_{i,t}$, higher the relative stability of a company in market performances.

From equation (3-28); $R_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t$

For a fitted model;

$$\begin{aligned} \bar{R}_t &= \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) \\ &= (a_1 \sin \omega t + b_1 \cos \omega t) + \dots + (a_n \sin n\omega t + b_n \cos n\omega t) \end{aligned} \tag{3-36}$$

According to (3-36), the motion comprises of several circular motions with radius $a_{i,}$ and $b_{i.}$ Hence $r_{i,t}$ was taken as the average of the radii;

$$r_i = (\sum_{i=1}^{n} | a_i | + | b_i |) / n \qquad (3\text{-}37)$$

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Statistical Software Used for the Analysis

The data analysis was focused to develop statistical models for forecasting Sri Lankan stock returns and to develop an indicator to measure the risk of returns of individual companies. Statistical software; MATLAB 2013b, MINITAB 17 and SPSS 21 were used in the data analysis.

Outliers of the data were adjusted by the MATLAB program given in Appendix 1.1 and the adjusted data were used in the analysis.

4.2 Pattern Recognition of Stock Returns

This study was focused on univariate time series analysis. Univariate statistical modeling procedure is based on the past internal patterns; as such pattern recognition was conducted to spot the suitable techniques for forecasting returns. Total Market (TM) returns, returns of twenty business sectors and returns of random sample of fifty companies were studied with the help of time series plots, histograms, box plots, normal probability plots, ACF and PACF. The Figure 4.1 is the time series plot of the total market returns.

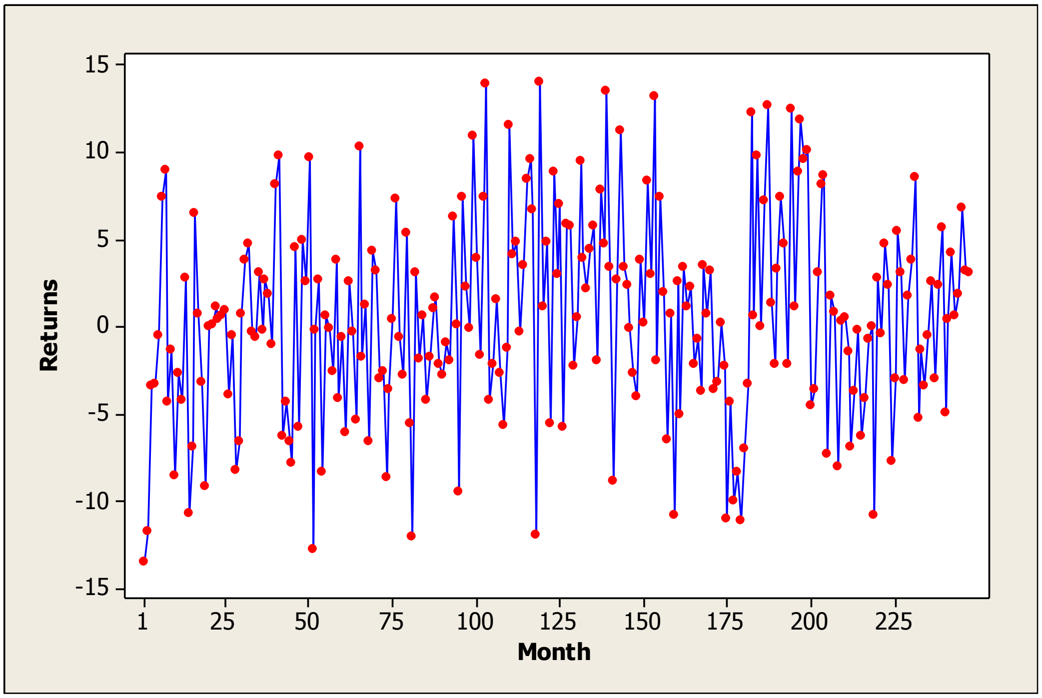
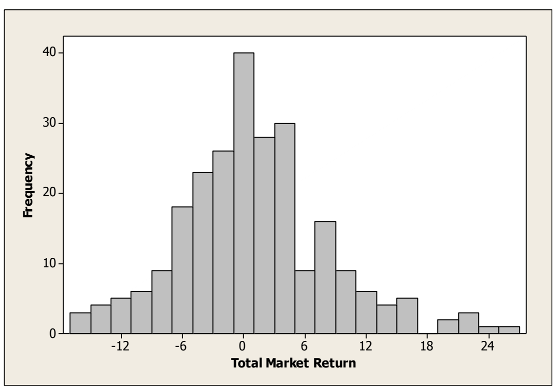


Figure 4.1: Time Series Plot of Total Market Returns

Figure 4.1 does not show any increasing or decreasing trend. Returns fluctuate between -15 and +15, shows a wave like pattern.



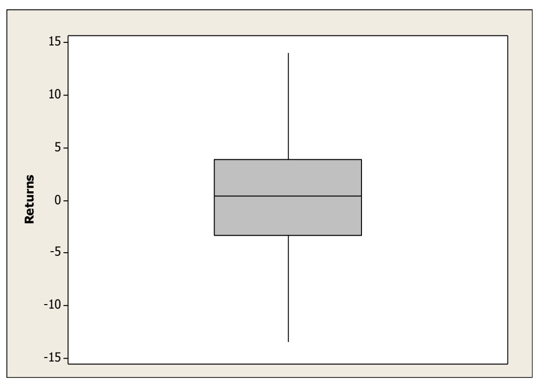


Figure 4.2: Histogram of TM Returns

Figure 4.3: Box- plot of TM Returns

Figure 4.2 is the Histogram and Figure 4.3 is the box-plot of Total Market(TM) returns. Figures show an almost symmetrical distribution for TM returns. But Probability plot of returns (Figure 4.4) and the P value of the Anderson Darling test (0.043 < 0.05) do not confirm the Normality of total market returns.

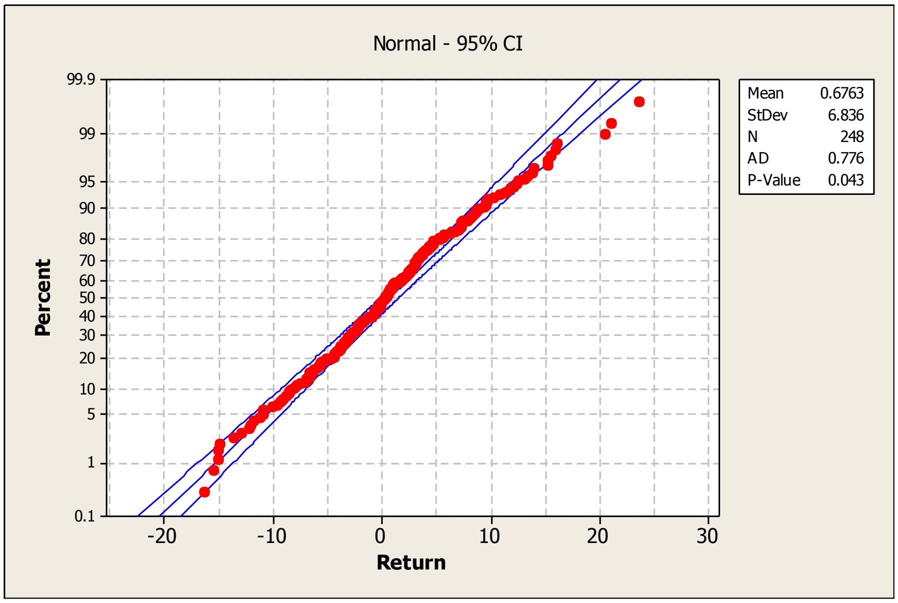


Figure 4.4: Probability Plot of Returns

Same procedure was repeated for business sectors and sample of companies. Time series plots are given in Appendix 2. According to the results; returns of majority of the cases were normally distributed. No increasing or decreasing trends were identified in stock returns of business sectors or individual companies. Returns of sectors as well as individual companies fluctuate within horizontal bands. Therefore it was concluded that the Sri Lankan stock market returns have wave like patterns.

4.3 Identification of Covariance Structure of Sri Lankan Share Market Returns

The literature review of the study gives empirical evidences for the incapability of the CAPM in forecasting returns. But, the CAPM model is a theoretically developed model. Therefore the principle assertion of CAPM, the linear relationship between the total market returns and the individual company returns cannot be easily rejected. Hence, the covariance structure of the Sri Lankan stock market returns was analyzed. Markowitz (1952) was interested in covariance between the individual company returns and the total market returns, but this study analyzed the; covariance between the sector returns and the total market returns, covariance between the individual company returns and the total market returns and covariance between the individual company returns and corresponding sector returns.

4.3.1 Covariance between Sector Returns and Total Market Returns

The covariance and correlation analysis were conducted between the sector returns and the total market returns. Summary of the outputs are given in the Table 4.1. The returns of the twenty business sectors were positively covariate with the returns of the total market. Returns of sectors BFI, DIV, BFT and MFG are strongly related to the market. The P-values correspond to the correlation analyses were less than the significance level (0.05). There exist a linear relationship between the sector returns and the total market returns.

Table 4.1: Summary of Covariance / Correlation Analysis between Sector Returns and Total Market Returns

Sector	Covariance	Pearson Correlation	P value
PLT	28.53	0.62	< 0.001
OIL	3.93	0.49	< 0.001
L&P	38.60	0.73	< 0.001
MTR	33.83	0.64	< 0.001
MFG	41.23	0.83	< 0.001
TLE	58.65	0.71	< 0.001
S&S	41.04	0.38	< 0.001
TRD	76.52	0.74	< 0.001
SRV	66.85	0.66	< 0.001
P&E	35.04	0.42	< 0.001
INV	88.86	0.63	< 0.001
H&T	54.25	0.77	< 0.001
HLT	36.33	0.53	< 0.001
F&T	60.10	0.62	< 0.001
IT	64.46	0.39	< 0.001
DIV	38.39	0.89	< 0.001
C&E	50.13	0.61	< 0.001
C&P	66.57	0.79	< 0.001
BFT	50.65	0.87	< 0.001
BFI	68.44	0.90	< 0.001

4.3.2 Covariance between Individual Company Returns and Total Market Returns

A simple random sample of twenty five companies was selected for the analysis. Summary of the analysis is given in Table 4.2.

Table 4.2: Summary of Covariance / Correlation Analysis between Individual Company Returns and Total Market Returns

Company	Covariance	Pearson Correlation	P value
EDEN	22.5	0.34	0.001
GHLL	3.01	0.05	0.460
PEGA	1.83	0.02	0.771
TAJ	26.7	0.44	< 0.001
TRAN	-0.62	-0.01	0.870
HUNA	26.41	0.40	0.001
PALM	5.83	0.09	0.371
SIGI	9.62	0.19	< 0.001
AHOT	35.51	0.50	< 0.001
AHUN	22.31	0.35	< 0.001
ABAN	13.83	0.17	0.011
ACL	5.34	0.09	0.190
KELA	0.14	0.02	0.841
COMB	-1.82	-0.03	0.622
DFCC	-3.57	-0.06	0.351
HNB	-1.63	-0.05	0.521
AGAL	1.86	0.47	0.000
BOGA	1.52	0.32	0.060
WATA	2.26	0.52	< 0.001
ACME	1.76	0.03	0.700
DISTIL	7.25	0.12	0.260
CLAND	-4.39	-0.06	0.541
KELSEY	-0.38	-0.03	0.770
TWOD	-0.57	-0.08	0.461
DIAL	0.34	0.01	0.990

Some of the companies have positive covariance with the total market returns, while the others have negative covariance with the total market returns. However, only 32% of the companies have significant linear relationship with the total market returns.

There are not enough evidences to conclude that the individual company returns move with the total market returns. This is contradictory to the idea of Markowitz (1952), the existence of linear relationship between the individual company returns and the total market returns. As such, inefficiency of the CAPM in forecasting individual company returns and incapability of β in measuring risk of individual company returns were confirmed for the Sri Lankan share market.

4.3.3 Covariance between Individual Company Returns and Sector Returns

A random sample of twenty five companies was used for the data analysis. Covariance and Pearson's correlation coefficients were obtained between individual company returns and sector returns. Summary of the analysis is given in Table 4.3. Returns of all the companies were positively covariate with the corresponding sector returns. Sample correlation coefficients for companies; DFCC, COMB and AGAL were strong, but weak for other companies. Still, P values of correlation analysis were less than the significance level (0.05) except for companies; PEGA, ACME and CLAND. It was concluded that the individual company returns and the corresponding sector returns are linearly related. Hence, it was attempted to forecast the individual company returns on corresponding sector returns.

Individual company returns are not associated with the total market returns, but associated with corresponding sector returns. It means individual company returns are not directly influenced by the macro level changes of the economy, but influenced by the micro level changes.

Table 4.3: Summary of Covariance / Correlation Analysis between Individual Company and Sector Returns,

Company	Covariance	Pearson correlation	P value
EDEN	33.2	0.40	0.001
GHLL	32.3	0.40	< 0.001
PEGA	8.6	0.09	0.291
TAJ	21.1	0.40	< 0.001
TRAN	6.4	0.13	0.060
HUNA	23.9	0.40	< 0.001
PALM	31.4	0.30	0.000
SIGI	27.4	0.20	0.010
AHOT	44.7	0.50	< 0.001
AHUN	37.9	0.40	< 0.001
ABAN	10.8	0.14	0.041
ACL	8.8	0.15	0.021
ACME	0.2	0.01	0.900
KELANI	3.2	0.54	0.001
SUGAR	2.3	0.51	< 0.001
COMB	38.2	0.77	< 0.001
DFCC	40.9	0.81	0.001
HNB	10.38	0.25	< 0.001
AGAL	3.73	0.72	< 0.001
BOGA	3.00	0.54	< 0.001
WATA	3.19	0.61	< 0.001
DISTIL	12.3	0.30	< 0.001
CLAND	7.22	0.10	0.351
KELSEY	6.81	0.49	0.001
TWOD	3.58	0.55	< 0.001

4.3.4 Model Individual Company Returns on Sector Returns

Individual company returns (R_t) were calculated by formula (3-1) and Sector returns (R_S) were calculated by formula (3-2). Formula (3-1) comprises share prices and formula (3-2) comprises monthly sector indices. Sector indices of CSE are market capitalization indices, calculated by the formula,

$$I_t = \frac{\sum_{i=1}^{n} P_{it}.Q_{it}}{\sum_{i=1}^{n} P_{i0}.Q_{i0}} \qquad (4\text{-}1)$$

Where, P_{it} = Price of stock *i* at time t, Q_{it} = Total outstanding shares for stock *i* at time *t*, P_{i0} = Price of stock *i* on base period and Q_{i0} = Total outstanding shares for stock *i* on base period.

Linear regression model is; $R_i = \beta_0 + \beta_1 R_S + \varepsilon$ (4-2)

Where R_i is the individual company return and R_S is the sector return. It was tested on sample of twenty companies. RMSE and MAD were used as measurements of errors. Two thirds of the data sets were used for model fitting and one third of the data set was used for model verification. Residual plots, ACF and PACF of residuals, Anderson Darling test and Durbin Watson test were used in residual analysis. Hypothesis test for regression coefficient is; H_0: β_1=0, H_1: $\beta_1 \neq 0$,

For example: fitted model for company GHLL is:

$$R_{GHLL} = -0.988 + 0.323 R_{H\&T} \qquad (4\text{-}3)$$

Where, R_{GHLL} is the returns of the company and $R_{H\&T}$ are the returns of the sector. P value of ANOVA (0.000) was less than the significance level (0.05), confirmed that the regression coefficient does not equal to zero. The RMSE and the MAD were small in model fitting and forecasting. The Durbin Watson test statistic (1.9557) was close to 2 confirmed the independence of residuals. The P-value of the Anderson Darling test (0.01) was less than the significance level (0.05); therefore normality of residuals was not confirmed. The Figure 4.5 shows that the pattern of forecasted returns is not similar to the pattern of actual returns. Hence, the fitted model was not recommended for forecasting.

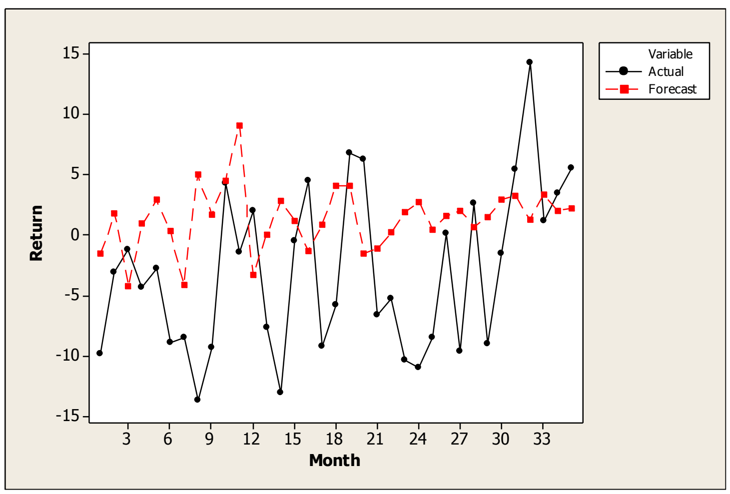


Figure 4.5: Actual Vs Forecasts-GHLL

Same procedure was repeated for the sample of companies. The Table 4.4 gives the summary of regression models of H&T;

Table 4.4: Summary of Regression Analysis- Sector H&T

Company	Model	Model Fitting		Model Verification		Remarks of residuals
		RMSE	MAD	RMSE	MAD	
GHLL	$R_{GHLL} = -0.988 + 0.323 R_{H\&T}$	8.0	6.0	8.4	7.0	Not normal, Independent
EDEN	$R_{EDEN} = 1.75 + 0.48 R_{H\&T}$	8.3	6.3	6.0	4.7	Not normal, Independent
PEGA	Model does not fit.					
TAJ	$R_{TAJ} = -0.16 + 0.29 R_{H\&T}$	9.3	7.0	7.2	5.6	Not normal, independent
TRAN	Model does not fit.					
HUNA	$R_{HUNA} = 0.04 + 0.34 R_{H\&T}$	7.7	6.1	5.3	3.8	Normal, Independent
PALM	$R_{PALM} = 0.29 + 0.45 R_{H\&T}$	9.2	7.2	9.2	7.5	Normal, Not independent
SIGI	$R_{SIGI} = 2.4 + 0.51 R_{H\&T}$	8.9	6.6	7.0	5.5	Normal, Independent
AHOT	$R_{AHOT} = 1.15 + 0.62 R_{H\&T}$	7.5	5.8	4.1	3.3	Not normal, Independent
AHUN	$R_{AHUN} = 0.82 + 0.49 R_{H\&T}$	7.9	6.1	4.7	3.5	Normal, Independent

For companies PEGA and TRAN, the P- values of ANOVA were greater than the significance level. As such the regression model does not fit. The RMSE and the MAD were small in all the fitted models. But the assumptions of the residuals were not satisfied in companies; GHLL, EDEN, TAJ, PALM and AHOT. As such the regression model was well fitted only for four companies.

The Table 4.5 gives the summary of regression analysis for the companies of the other sectors. For company KELA, the P- value of ANOVA was greater than the significance level. As such the regression model does not fit. For the companies; COMB and WATA, the assumptions of the residuals were not met. Accordingly, the method is successful only in the eleven out of the twenty companies.

In the above analysis, individual company returns were regressed on sector returns. These sector returns were calculated by using the corresponding sector indices. As such, the reliability of this method depends on the reliability of sector indices. One of the weaknesses identified in market capitalization indices are the weighting system. In these indices, total outstanding shares of a company are the weights. This allows the price movements of large companies to have a greater impact on the index (Samarakoon, 2010). Therefore forecasting individual company returns on corresponding sector returns is not successful.

Table 4.5: Summary of Regression Analysis- Sectors BFI, PLT, MFG, BFT, L&P, TLE

Sector	Company	Model	Model Fitting		Model Verification		Remarks of residuals
			RMSE	MAD	RMSE	MAD	
BFI	COMB	$R_{COMBANK} = 0.06 + 0.88 R_{BFI}$	5.0	3.4	4.4	2.9	Not normal or independent

	DFCC	$R_{DFCC} = -1.05 + 1.11 R_{BFI}$	4.5	3.2	5.1	3.6	Normal, Independent
	HNB	$R_{HNB} = 0.66 + 0.47 R_{BFI}$	6.6	5.4	5.4	5.4	Normal, Independent
PLT	AGAL	$R_{AGAL} = -0.13 + 0.06 R_{PLT}$	0.4	0.4	0.4	0.3	Normal, Independent
	BOGA	$R_{BOGA} = -0.08 + 0.06 R_{PLT}$	0.5	0.4	0.6	0.4	Normal, Independent
	WATA	$R_{WATA} = 0.14 + 0.05 R_{PLT}$	0.4	0.3	0.6	0.6	Not normal or Independent
MFG	KELA	Model does not fit.					
BFT	DISTIL	$R_{DIST} = 0.92 + 0.40 R_{BFT}$	7.3	5.8	5.2	4.2	Normal, Independent
L&P	KELSEY	$R_{KELS} = 0.34 + 0.93 R_{L\&P}$	7.9	6.1	7.4	6.0	Normal, Independent
TLE	DIAL	$R_{DIAL} = -1.09 + 0.39 R_{TEL}$	5.1	4.4	4.4	3.8	Normal, Independent

4.4 ARIMA Models on Forecasting Returns

The ARIMA model was tested on sample of fifty companies, representing ten business sectors. Monthly percentage returns were used for the analysis. Stationary of the series were tested by: ACFs, PACFs and the Augmented Dickey Fuller Test (ADFT). A part of the ADFT test results are given in Appendix 3.1. Two thirds of a data set was used for model fitting and one third of the data set was used for model verification. The histogram of residuals, normal probability plot of residuals and the Anderson Darling test were used to test the normality of residuals. The graph of residual Vs fits, the ACF and the PACF of residuals and the LBQ statistics were used to test the independence of residuals. Graphs of actual Vs forecasts were obtained to see whether the forecasted returns follow the pattern of actual returns for the verification period.

Table 4.6 is the summary of best fitted models of the sector H&T. One or two of the assumptions of the residuals were not satisfied by four of the models; TAJ, TRAN, AHOT and AHUN. As such they are not valid models for forecasting. Measurements of errors were small in all the well fitted models, in both model fitting and verification. Therefore ARIMA models are successful in forecasting returns of the individual companies of sector H&T. But, the pattern of forecasted returns was linear, different from the patterns of actual returns.

Table 4.6: Summary of ARIMA Models for Sector H&T

Company	Best Fitting ARIMA (p,d,q) Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
EDEN	(0,1,1)	8.2	6.8	7.8	6.3	Normal, independent
GHLL	(0,0,1)	8.1	6.4	7.5	6.3	Normal, independent

PEGA	(0,1,1)	9.1	7.2	6.8	5.4	Normal, independent
TAJ	(0,1,1)	8.4	6.7	7.4	6.3	Not normal / not independent
TRAN	(0,1,1)	9.1	5.7	7.2	5.2	Normal / not independent
HUNA	(0,1,1)	7.2	5.8	3.6	2.8	Normal, independent
PALM	(0,1,1)	7.4	5.7	8.4	6.7	Normal, independent
SIGI	(0,1,1)	7.5	6.8	4.6	3.5	Normal, independent
AHOT	(0,1,1)	6.8	5.7	6.1	4.9	Not normal / not independent
AHUN	(0,1,1)	8.1	5.7	7.2	5.2	Not normal / not independent

Table 4.7 is the summary of best fitted models of the sector MFG. Assumptions of the residuals were not satisfied by four of the fitted models; as such they are not valid models for forecasting. Measurements of errors were small in all the fitted models, both model fitting and verification. The ARIMA (0,1,1) model was fit in to seven out of eleven companies of sector MFG. But, the pattern of forecasted returns was linear, different from the patterns of actual returns.

Table 4.7: Summary of ARIMA Models- Sector MFG

Company	Best Fitting ARIMA (p,d,q) Model	Model Fitting	Model Verification	Remarks of Residuals

		RMSE	MAD	RMSE	MAD	
ABAN	(0,1,1)	5.9	4.5	7.8	6.4	Normal, independent
ACL	(0,1,1)	7.3	6.9	6.8	5.4	Normal / not independent
ACME	(0,1,1)	8.4	6.7	9.3	8.1	Normal, independent
KELA	(0,1,1)	8.3	5.9	6.8	5.4	Normal, not independent
LMF	(0,1,1)	8.3	6.9	6.8	5.4	Normal, independent
TOKY	(0,1,1)	6.7	5.2	5.8	4.9	Normal, independent
WALL	(0,1,1)	7.1	5.9	9.0	6.6	Normal, independent
ROCEL	(0,1,1)	8.8	7.9	6.8	6.4	Not normal / not
BLUE	(0,1,1)	9.3	7.1	9.7	7.7	Normal, independent
BOGAL	(0,1,1)	8.8	6.5	8.4	7.3	Normal, independent
CERA	(0,1,1)	8.4	6.4	5.5	4.4	Not Normal or independent

Summary of ARIMA Models for Sector BFI is given below in Table 4.8;

Table 4.8: Summary of ARIMA Models- Sector BFI

Company	Best Fitting ARIMA (p,d,q) Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
ALLI	(0,1,1)	6.9	5.3	7.3	6.4	Normal, independent

ASIA	(0,1,1)	9.1	7.3	8.8	6.5	Normal, independent
COMB	(0,1,1)	8.3	6.5	4.7	3.7	Normal, independent
DFCC	(0,1,1)	8.4	6.6	6.5	2	Normal, Independent
HNB	(0,0,1)	5.2	4.4	4.6	3.6	Normal, independent
LFIN	(0,1,1)	8.1	6.6	6.3	5.7	Normal, independent
LOLC	(0,1,1)	9.8	7.7	9.1	7.3	Normal, independent
SAMP	(0,1,1)	7.8	6.1	6.7	5.6	Normal, independent
HASU	(0,0,1)	6.6	5.3	5.5	0.3	Normal, independent

The Bank Finance and Insurance (BFI) sector is considered as the most important sector of the economy of a country. From the point of view of capital market experts, the sector BFI of the Sri Lankan share market is highly volatile and unpredictable. But the results in Table 4.8 do not agree with the claim. It shows that the ARIMA model is successful in forecasting returns of the sector BFI. However, the pattern of forecast was different from the actual returns.

The business sector L&P of the CSE has twenty two companies. The sample consist five companies and the ARIMA model was successful in four of them. Forecasting errors were small in all the fitted models.

Table 4.9: Summary of ARIMA Models- Sector L&P

Company	Best Fitting ARIMA (p,d,q) Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
CLAND	(0,1,1)	8.0	6.2	6.5	5.1	Normal, independent
KELSEY	(0,0,1)	7.9	6.3	8.7	7.0	Not normal, independent
PDL	(0,1,1)	5.9	4.8	5.4	4.5	Normal, independent
EAST	(0,1,1)	11.3	9.0	9.8	7.8	Normal, independent
EQIT	(0,0,1)	8.9	6.3	8.7	7.2	Normal, independent

Eighteen companies were listed in the Plantation sector of the CSE. The sample of the study contains four of them. According to Table 4.10, forecasting errors of the fitted models were small in both model fitting and verification. Residuals of the models were independent and normally distributed. Therefore the ARIMA (0, 1, 1) model is suitable for forecasting returns of individual companies of the sector PLT. . However, the pattern of forecast was different from the actual returns.

Table 4.10: Summary of ARIMA Models- Sector PLT

Company	Best Fitting ARIMA (p,d,q) Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
AGAL	(0,1,1)	8.2	6.6	6.8	5.9	Normal, independent
BALA	(0,1,1)	8.7	6.7	8.2	6.2	Normal, independent
BOGA	(0,1,1)	9.6	7.6	7.6	6.3	Normal, independent
WATA	(0,1,1)	7.6	5.3	7.2	5.9	Normal, independent

The Table 4.11 summarizes the ARIMA outputs for sectors; BFT, DIV, C&P, MTR and TLE. The ARIMA model was well fitted for; two companies of the sector BFT, two companies of the sector DIV, and two companies of the sector C&P and so on.

Table 4.11: Summary of ARIMA - Sectors BFT, DIV, C&P, MTR and TLE

Sector	Company	Best Fitting ARIMA (p,d,q) Model	Model Fitting		Model Verification		Remarks of Residuals
			RMSE	MAD	RMSE	MAD	
BFT	BREW	(0,1,1)	8.3	6.8	6.5	5.4	Normal, independent
	DISTIL	(0,0,1)	8.4	7.3	6.2	4.7	Normal, not dependent
	NESTL	(0,1,2)	4.6	3.6	4.3	3.2	Normal, independent
DIV	HAYL	(0,1,1)	7.5	5.9	6.5	5.9	Normal, independent
	JKH	(0,1,1)	7.4	6.3	5.2	3.7	Normal, Independent
	RICHA	(0,1,1)	8.8	5.3	7.6	4.9	Not normal, independent
C&P	LANK	(0,0,1)	10.3	8.0	8.5	7.0	Normal, independent
	CIC	(0,0,1)	7.7	6.0	8.6	7.3	Normal, independent
MTR	DIMO	(0,0,1)	6.8	5.3	6.6	4.9	Normal, independent
	AMW	(0,0,1)	6.9	6.3	6.6	5.9	Not normal, dependent
TLE	DIAL	(0,1,1)	5.0	3.9	8.8	7.5	Normal, Independent

The ARIMA model was suitable for forecasting 78% of the sample of companies. The ARIMA (0, 1, 1) was the mostly fitted model. It is clear that the stock returns of the Sri Lankan companies depend on past errors.

Literature gives evidence for the forecasting ability of the ARIMA in stock returns for several countries. Result of the present study also confirms the same for the Sri Lankan context. However, forecasted values of all the models followed linear patterns, not the patterns of the actual returns. Ayodele et al. (2014-b) also pointed out the same weakness in forecasting share prices. This is a clear disadvantage of the method.

4.5 Fourier Transformation on Forecasting Stock Returns

Fourier transformation is a linear transformation. A transformation moves all the points in (x, y) plane according to some rule. A special property of the linear transformation is that, it involves only linear expressions of x and y (Attwood, Cope, Moran, Pateman, Pledger, Staley and Wilkins, 2008). Rotations, reflections and enlargements are examples for linear transformation. The Fourier transformation employs rotation. The discrete version of the Fourier transformation, given in formula (3-24) is a static model. It has been applied to explain the regular waves in Physics. It is modified to describe the wave patterns associated with randomness as shown in formula (3-28);

$$R_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t$$

The model (3-28), which is named as the "Circular Model (CM)" was tested on sample of fifty companies. Amplitudes of the series, a_k and b_k are experimentally calculated in Physics. As it is not possible in the applications, this study employs multiple regression technique for the purpose; regressing R_t on $sink\omega t$ and $cosk\omega t$ for k is from 1 to 6.

For example, the angular speed (ω) for the company EDEN was calculated by,

$$\omega = 2\pi f / N ,$$

Where f is the number of peaks and N is the number of observations in the series.

For EDEN, f=45 and N=200; hence ω=1.4137. Part of the Fourier transformed data is given in the Appendix 4.1. The correlation analysis confirmed the independence of these series. Hence, R_t was regressed on them.

$$R_t = -0.58047 + 1.8205 \sin 4\omega t + 1.6831 \cos 3\omega t - 2.1024 \cos 4\omega t \qquad (4\text{-}4)$$

The RMSE and the MAD for the model (4-4) were small. Residuals of the model were normally distributed and independent. Therefore the Circular Model is suitable in forecasting returns of the company EDEN. Same procedure was repeated for the other companies. Two thirds of the data sets were used for model fitting and one third of the data set was used for model verification. The Circular Model was well fitted for sector L&P. Summary results are given in Table 4.12;

Table 4.12: Circular Model in Forecasting Returns of Sector H&T

Company	Best Fitting Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
EDEN	$R_t = -0.58047 + 1.8205\sin 4\omega t + 1.6831\cos 3\omega t - 2.1024\cos 4\omega t$	7.9	6.3	5.7	4.4	Normal, Uncorrelated
GHLL	$R_t = -1.4949 - 0.805\cos 4\omega t$	8.17	6.4	6.7	5.7	Normal, Uncorrelated
PEGA	$R_t = -1.4882 + 2.1316\cos \omega t$	8.91	7.15	7.07	5.67	Normal, Uncorrelated
TAJ	$R_t = -0.6397 - 1.2099\sin 4\omega t + 1.465\sin 5\omega t + 1.2075\cos 4\omega t$	5.87	4.61	5.79	4.36	Normal, Uncorrelated
TRAN	$R_t = 0.5653 - 1.5104\sin \omega t$	7.33	5.7	5.97	4.67	Normal, Uncorrelated
HUNA	$R_t = 0.34407 - 2.5198\cos \omega t$	6.95	5.58	3.46	2.10	Normal, Uncorrelated
PALM	$R_t = 1.6629 - 3.6778\cos 5\omega t$	8.6	6.7	8.8	7.0	Normal, Uncorrelated
SIGI	$R_t = 0.33287 + 3.0827\sin 3\omega t$	7.47	6.02	4.07	3.36	Normal, Uncorrelated
AHOT	$R_t = 0.85139 + 1.6217\sin 4\omega t$	6.46	5.50	4.36	3.47	Normal, Uncorrelated
AHUN	$R_t = 1.3307 - 1.9955\sin 5\omega t$	8.61	7.06	5.84	4.40	Normal, Uncorrelated

The fitted model for EDEN, given in (4-4) comprises three trigonometric functions; *sin4ωt, cos3ωt* and *cos4ωt*. In other words the motion of returns comprises three circular motions; circle C1 with angular speed *4ωt* and radius 1.8205, circle C2 with angular speed *3ωt* and radius 1.6831and circle C3 with angular speed *4ωt* and radius 2.1024. Waves related to circular motions C1, C2 and C3 are given in Figure 4-3, Figure 4-4, and Figure 4-5 respectively;

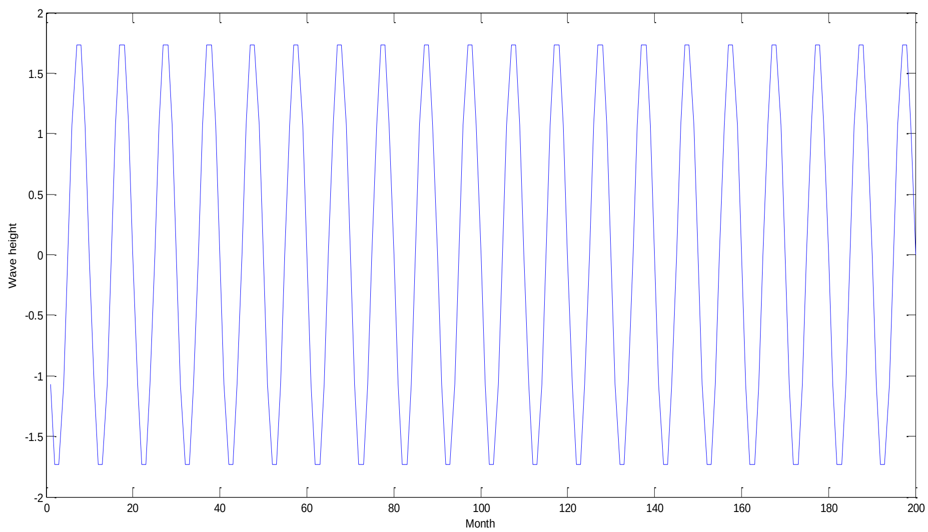


Figure 4.6: Plot of Wave C1

The wave C1 is a regular wave with the amplitude 1.8205 and the period of the oscillation fifteen months. The wave C2 is an irregular wave with highest amplitude 1.6831 and the period of the oscillation five months.

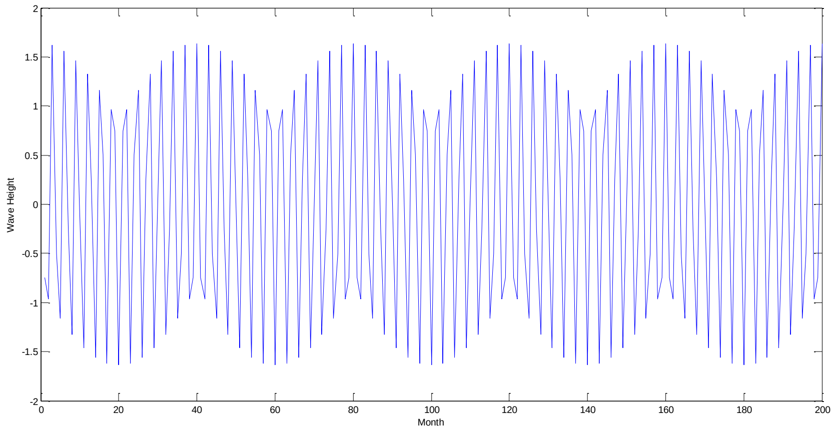


Figure 4.7: Plot of Wave C2

The wave C3 is a regular wave with amplitude 2.1024 and the period of the oscillation sixteen months.

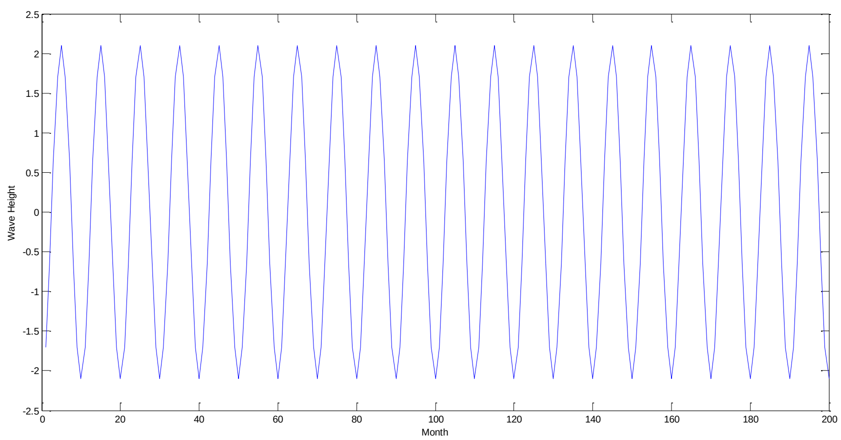


Figure 4.8: Plot of Wave C3

The sector MFG has thirty one companies and the sample contained eleven of them. The Circular Model (CM) was fitted for ten of them.

Table 4.13: Circular Model in Forecasting Returns of Sector MFG

Company	Best Fitting Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
ABAN	$R_t = 1.9194 + 1.7358 \cos 5\omega t$	5.7	4.5	6.7	5.2	Normal, Uncorrelated
ACL	$R_t = -0.15825 - 1.47 \sin 4\omega t$	7.34	5.88	4.80	3.69	Normal, Uncorrelated
ACME	$R_t = -0.41894 - 2.7181 \sin 4\omega t$	8.21	6.62	9.68	8.24	Normal, Uncorrelated
KELA	$R_t = 0.13136 + 2.8135 \sin 3\omega t - 2.3953 \cos 5\omega t$	8.52	6.88	6.92	5.32	Normal, Uncorrelated
TOKY	$R_t = 0.26593 - 1.34 \sin 5\omega t - 1.3662 \cos 5\omega t$	6.62	5.02	5.92	4.94	Normal, Uncorrelated
WALL	$R_t = 1.1735 - 2.727 \sin 5\omega t$	7.05	5.57	9.05	7.31	Normal, Uncorrelated
ROCEL	$R_t = 1.4475 + 2.4641 \sin 3\omega t - 3.9941 \cos 5\omega t$	7.8	6.42	8.57	7.12	Normal, Uncorrelated
BLUE	$R_t = -2.3482 + 2.5493 \cos 3\omega t$	8.88	6.87	9.63	7.89	Normal, Uncorrelated
BOGAL	$R_t = -0.7824 + 2.3738 \sin \omega t + 2.3434 \cos \omega t + 2.9714 \cos 4\omega t$	7.47	6.17	7.73	6.38	Normal, Uncorrelated
CERA	$R_t = 0.2826 - 2.674 \sin 5\omega t$	8.1	6.4	5.9	4.6	Normal, Uncorrelated

Table 4.14: Circular Model in Forecasting Returns of Sector BFI

Company	Best Fitting Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
ALLI	$R_t = 1.4899 + 0.6092\cos\omega t$	7.07	5.45	6.85	5.34	Normal, Uncorrelated
ASIA	$R_t = -0.30087 + 2.9082\sin 4\omega t$	8.9	7.1	8.75	7.18	Normal, Uncorrelated
DFCC	$R_t = 0.31742 - 2.0337\cos 5\omega t$	8.3	6.5	6.06	4.96	Normal, Uncorrelated
HNB	$R_t = 0.77154 - 1.601\sin 5\omega t - 1.2866\cos 5\omega t$	5.7	4.4	4.3	3.5	Normal, Uncorrelated
LFIN	$R_t = -0.21742 + 2.3374\sin 6\omega t + 2.11110.\cos\omega t$	7.89	6.48	5.76	4.35	Normal, Uncorrelated
LOLC	$R_t = 1.0022 + 1.9094\cos 2\omega t$	9.7	7.6	8.8	6.8	Normal, Uncorrelated
SAMP	Model does not fit					
HASU	$R_t = 1.0822 - 2.1016\sin 5\omega t$	6.7	5.3	5.7	4.4	Normal, Uncorrelated

The Bank Finance and Insurance (BFI) sector is considered as the most important sector of the economy of a country. From the point of view of capital market experts, the sector BFI of the Sri Lankan share market is highly volatile and unpredictable. According to the results in Table 4.14, the Circular Model was successful in forecasting returns of seven out of eight companies of the sector BFI. Patterns of the forecasts follow the patterns of actual returns. For the company SAMP, none of the regression coefficients were significantly different from zero. As such the CM was not fitted for the company SAMP.

The Table below summarizes the analysis of the sector L&P. The CM did not fit into the returns of the company KELS, as none of the regression coefficients were significantly different from zero. But the CM was well fitted for returns of other four companies.

Table 4.15: Circular Model in Forecasting Returns of Sector L&P

Company	Best Fitting Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
CLAND	$R_t = -0.930 - 1.476 \cos 6\omega t$	7.8	6.1	6.9	5.6	Normal, Uncorrelated
KELS	Model does not fit					
PDL	$R_t = 0.1549 - 1.4199 \sin 5\omega t - 1.4872 \cos 6\omega t$	5.7	4.6	5.2	4.1	Normal, Uncorrelated
EAST	$R_t = -0.34951 - 4.3751 \sin 5\omega t$	10.1	7.9	9.3	7.1	Normal, Uncorrelated
EQIT	$R_t = 0.5024 - 2.1354 \sin \omega t$	7.9	6.4	9.1	7.2	Normal, Uncorrelated

The Table 4.16 summarizes the results for sectors; PLT, BFT, DIV, C&P, MTR and TLE. The model did not fit to the company HAYL of the sector DIV; company CIC of the sector C&P; Company AMW of the sector MTR and the company DIAL of the sector TLE.

Table 4.16: Circular Model for Sectors; PLT, BFT, DIV, C&P, MTR and TLE

Sector	Company	Best Fitting Model	Model Fitting		Model Verification		Remarks of Residuals
			RMSE	MAD	RMSE	MAD	
PLT	AGAL	$R_t = -0.5 + 2.1\sin 3\omega t - 2.5.\sin 5\omega t$	8.8	7.1	7.8	5.9	Normal, Uncorrelated
	BALA	$R_t = 0.07576 - 2.5256\sin 3\omega t$	8.54	6.7	8.26	6.04	Normal, Uncorrelated
	BOGA	$R_t = 0.2733 - 3.1889\cos 3\omega t$	9.3	7.3	8.6	6.7	Normal, Uncorrelated
	WATA	$R_t = -1.316 - 2.245\cos 6\omega t$	7.9	6.3	6.6	5.3	Not Normal, Uncorrelated
BFT	BREW	$R_t = 1.665 + 2.4188\sin 4\omega t$	8.3	6.6	6.0	4.6	Normal, Uncorrelated
	DISTIL	$R_t = 0.0179 + 1.47\cos 6\omega t$	6.96	5.57	4.59	3.61	Normal, Uncorrelated
	NESTL	$R_t = 1.5412 - 1.4092\cos 3\omega t$	4.44	3.51	4.11	3.21	Normal,
DIV	JKH	$R_t = 0.43 + 2.2917\cos 3\omega t$	7.35	5.88	4.53	3,43	Normal, Uncorrelated
	RICHA	$R_t = -0.359 + 1.5138\cos 5\omega t$	7.35	5.93	5.43	4.53	Normal, Uncorrelated
C&P	LANK	$R_t = 0.052 + 2.114\sin\omega t$	10.5	8.3	8.8	7.3	Normal, Uncorrelated
MTR	DIMO	$R_t = -0.4712 - -1.3696\sin\omega t$	7.0	5.4	6.8	5.0	Normal, Uncorrelated

The Circular model was successful in 82% of the companies of the sample. Patterns of the forecasted returns followed the patterns of actual returns. As such Circular Model is suitable in forecasting individual company returns of the Sri Lankan stock market.

4.6 Test the ARCH effect on Returns

The Engle's ARCH test was used to check whether the ARCH effect exist in individual company returns. A simple random sample of thirty companies was used for testing the hypotheses;

H_0: There is no ARCH up to order 1 in returns

H_0: There is ARCH up to order 1 in returns

The MATLAB syntax, [h,p,fStat,crit] = archtest(e,'Lags',m) was used. The result h = 1 indicates the rejection of null hypothesis of no conditional heteroscedasticity and conclude that there are significant ARCH effects in the series. At 5% significance level, the Table value (F table) is 3.8415. The summary of the analysis is given in Appendix 3.2 For the company PEGA, the F statistics (1.2392) or the test statistics value is less than the critical value of the F table (3.8415). Therefore the null hypothesis is not rejected and, concluded that the ARCH effect does not exist in the return series. Conversely; for the company GHLL, the F statistics (10.9576) is greater than the critical value of the F table (3.8415). Therefore the null hypothesis is rejected and, concluded that the ARCH effect exists in the return series. Accordingly, ARCH effect does not exist in 80% of the return series. As such the ARCH model cannot be used to measure the risk of returns.

4.7 Make Circular Indicator

This part of the study was based on the Newton's law of uniform circular motion. The law is widely applied in explaining; satellites orbiting the Earth, planets orbiting the Sun, motion of a vehicle in a circular path, motion in a banked track, playground Merry-go-Rounds etc.

Individual company returns of the Sri Lankan share market move in circular paths. In stock market performances, market demand would be the centripetal force. The measurement, named as the Circular Indicator (*CI*) was calculated by using the formula

$$F_{i,t} = r_{i,t} . \omega_{i,t}^2$$

For example; the Circular Model for the company EDEN is;

$$R_t = -0.58047 + 1.8205 \sin 4\omega t + 1.6831 \cos 3\omega t - 2.1024 \cos 4\omega t$$

The average amplitude of the wave, *r* is the average of the radii of the reference circles;

$$\begin{aligned} r &= (1.8205 + 1.6831 \ + 2.1024)/3 \\ &= 1.8686 \end{aligned}$$

Hence, the *CI* for the company EDEN is 4.936.

The Circular Indicators for year 2014 were calculated for individual companies; given in Tables 4.17 to 4.19, sector wise.

Table 4.17: Circular Indicators (*CI*) for Returns- Sector H&T

Company	Angular Speed (ω)	Amplitude (r^*)	Circular Indicator (*CI*)
EDEN	1.413	1.868	4.936
GHLL	1.361	1.805	4.435
PEGA	1.298	2.131	5.900
TAJ	1.413	1.587	3.560
TRAN	1.476	1.510	3.368
HUNA	1.570	2.519	9.973
PALM	1.335	3.677	18.060
SIGI	1.466	3.0827	13.932
AHOT	1.549	1.621	4.076
AHUN	1.172	1.995	4.670

The company PALM has the highest *CI* and the company TRAN has the lowest *CI*. Accordingly, the lowest level of risk is in corporate with the company PALM and the highest level of risk is in corporate with the company TRAN. In other words, most stable company of the sector L&P in share market performances is PALM and the least stable company is TRAN.

Table 4.18: Circular Indicators (*CI*) for Returns- Sector MFG

Company	Angular Speed (ω)	Amplitude (r^*)	Circular Indicator (C_I)
ABAN	1.529	1.735	4.609
ACL	1.288	1.470	2.7835
ACME	1.466	2.718	10.831
KELA	1.2147	2.6044	8.2392
TOKY	1.3509	1.3531	2.4733
WALL	1.178	2.727	8.761
ROCEL	1.319	3.229	13.755
BLUE	1.445	2.549	9.391
BOGAL	1.466	3.153	14.579
CERA	1.211	2.674	8.664

Results of the Table 4.18 revealed that the companies BOGAL, ROCEL and ACME are highly stable. Circular indicators for sectors; BFI, PLT, BFT, DIV, L&P, C&P and MTR are given in the Table 4.19. Accordingly, the most stable company for sector BFI is ASIA, for sector PLT is BOGA and so on.

Table 4.19: Circular Indicators for Sectors BFI, PLT, BFT, DIV, L&P, C&P and MTR

Sector	Company	Angular Speed (ω)	Amplitude (r^*)	Circular Indicator (*CI*)
BFI	ALLI	1.501	1.735	4.522
	ASIA	1.221	2.908	10.332
	DFCC	1.282	2.033	5.305
	HNB	1.282	2.053	5.411
	LFIN	1.413	2.224	6.994
	LOLC	1.340	1.909	4.886
	HASU	1.382	2.101	6.105
PLT	AGAL	1.217	2.359	6.777
	BALA	1.691	2.525	10.790
	BOGA	1.605	3.188	16.328
	WATA	1.267	2.245	6.392
BFT	BREW	1.546	2.418	9.048
	DISTIL	1.193	1.479	2.612
	NESTL	1.424	1.409	2.828
DIV	JKH	1.424	2.291	7.479
	RICHA	1.472	1.513	3.373
L&P	CLAND	1.466	1.476	3.194
	PDL	1.498	1.453	3.165
	EAST	1.363	4.375	26.105
	EQIT	1.112	2.135	5.068
C&P	LANKEM	1.2828	2.111	5.718
MTR	DIMO	1.393	1.3696	2.613

Chapter Summary

Stock returns of the Sri Lankan share market follow wave like patterns. They do not have any trend or seasonal components. The covariance analysis revealed that the individual company returns move with the corresponding sector returns, but not with the total market returns. Forecasting share returns based on the covariance structure of the market was not highly successful.

The most important sector for the economy of a country is the Bank Finance & Insurance (BFI). It is believed that the sector BFI is highly volatile; as such the returns are unpredictable. But this study found two successful methods for the purpose; ARIMA and CM. The Hotel & Travel (H&T) is the third income generator of the country (Konarasinghe, 2015). It is a fast growing industry after the thirty years of civil war. Hence the investor's attraction towards the sector is increasing. The study found both ARIMA and CM as suitable techniques for forecasting individual company returns of the industry.

The Sri Lankan stock market comprises twenty business sectors. The sample of the study contained fifty companies, representing ten of them. The ARIMA technique was successful in 78% of the companies and the CM was successful in 82% of the companies. The RMSE and the MAD were equally small in both techniques. However, the forecasted values of ARIMA models did not follow the patterns of the actual returns, but the forecasted values of CM followed the actual returns. The Figure 4.6, the actual returns, ARIMA forecasts and CM forecasts for the company HUNA is an example for patterns of actual returns Vs ARIMA and CM. The graphs for some other companies are given in

Appendix 5. Accordingly, the CM is superior to ARIMA in forecasting individual company returns of the Sri Lankan share market.

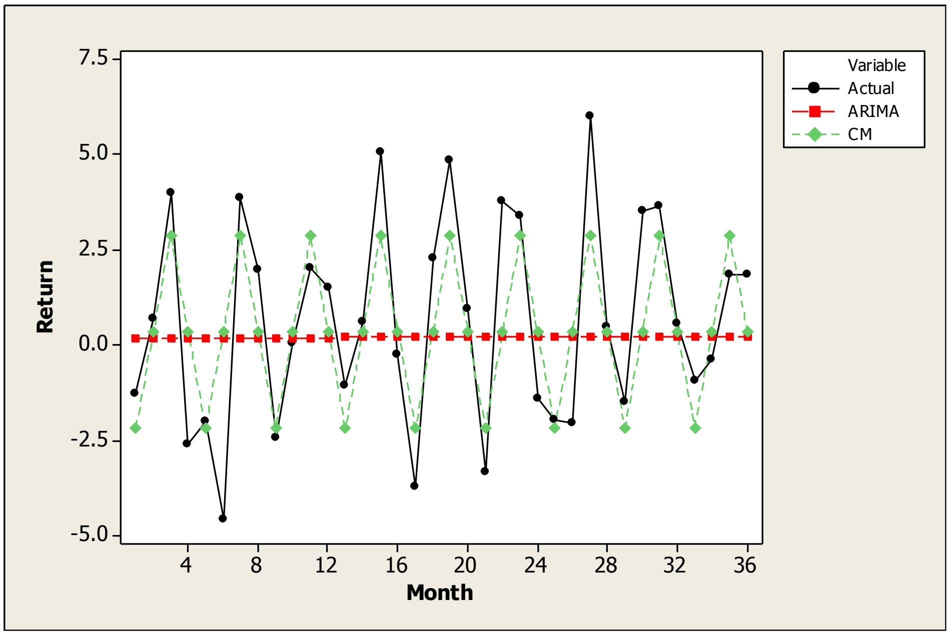


Figure 4.9: Actual Returns Vs ARIMA & CM Forecasts

The Circular Indicator is a theoretically developed method for measuring risk of returns. Therefore, it is a better measurement than the existing risk measurements; the standard deviation of returns and the beta (β) coefficient of CAPM.

CHAPTER 5

CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

The study was carried out with two objectives; development of forecasting models for Sri Lankan stock returns and the development of stability indicator for market performances of individual companies in the Sri Lankan share market,. The study was begun with pattern recognition of stock returns. It was concluded that; the total market returns, the sector returns and the individual company returns of CSE follows wave like patterns without any trend or seasonal pattern. Model development of the study was based on Technical Analysis. The mostly used technical analysis based model was CAPM, but many researchers have challenged the central assertion of CAPM; existence of linear relationship between the returns and the risk. As such the analysis was begun with the identification of covariance structure of the Sri Lankan share market. It was concluded that; the sector returns move with the total market returns, and individual company returns move with the corresponding sector returns. But there was no significant relationship between individual company returns and the total market returns. It was concluded that the central assertion of CAPM is invalid for the Sri Lankan share market. In Sri Lankan stock market, the risk of returns is measured by β coefficient of CAPM. The study evidenced that the CAPM does not hold for Sri Lankan share market, hence β coefficient is not a suitable measurement for risk of returns.

The linear relationship between the individual company returns and the sector returns were used for model fitting. It was concluded that the simple linear regression model of individual company returns on sector returns is not suitable for forecasting returns.

Returns of all the companies were stationary type; hence the ARIMA model was tested on forecasting individual company returns. It was concluded that the ARIMA models are suitable in forecasting individual company returns of Sri Lankan share market. Then the Circular Model was tested on returns and found that the model is successful in 84% of the sample of companies. Finally, it was concluded the Circular Model is the best forecasting model for the Sri Lankan share market. Also, it was concluded that the Circular Indicator is a suitable measurement for the risk of returns.

5.2 Recommendations

The CAPM suggests a linear relationship between an individual company return and the market risk. The assertion was rejected for the Sri Lankan context. Still, it was found that the sector returns are linearly related to total market returns, and the individual company returns linearly related to sector returns. As such, there exist a relationship between individual company return and the total market return. It is recommended to use the theory of relative motion to identify the hidden relationship, in order to improve CAPM.

Wave like patterns is a common feature in natural sciences. The Circular Model may be applicable in the fields of Agriculture, Medicine, Meteorology and many others. Especially, the Circular Indicator may be highly useful in risk assessments. It is recommended to apply the Circular Model and the Circular Indicator for more real life situations in different fields.

5.3 List of Publications of the Study

Twenty two research articles were published from the study; twelve in peer reviewed, indexed journals and ten in conference proceedings;

Referred Journals

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2016). Circular Methods on Forecasting Risk & Returns of Share Market Investments. *International Journal of Research & Review*. 3(4), 51-56, Available at, www.gkpublication.in

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2016). Circular Model on Forecasting Returns of Sri Lankan Share Market. *International Journal of Novel Research in Physics, Chemistry and Mathematics*, 3(1), 49-56. Available at: www.noveltyjournals.com

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2016). Comparison of ARIMA & Circular Model in Forecasting Sri Lankan Share Market Returns. *International Journal of Novel Research in Interdisciplinary Studies*, 3(2), 3-8. Available at: www.noveltyjournals.com

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2016). Spectral Analysis on Forecasting Sri Lankan Share Market Returns. *International Journal of Management & Applied Science*. 2(1), 25-28. Available at, http://ijmas.iraj.in/

Konarasinghe, W.G.S. (2016). Circular Indicator on Measuring Risk of Returns of Sri Lankan Share Market. *Global Journal for Research Analysis*, 5(4), 247-248. Available at: http://theglobaljournals.com/

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2015). Forecasting Share Prices of Banks of Colombo Stock Exchange. *International Journal of Research & Review*. 2(6), 372-378, Available at, www.gkpublication.in

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2015). Comparison of Time Domain and Frequency Domain Analysis in Forecasting Sri Lankan Share Market Returns. *International Journal of Management & Applied Science.* 1(9),16-18, Available at, http://ijmas.iraj.in/

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2015). ARIMA Models on Forecasting Sri Lankan Share Market Returns. *International Journal of Novel Research in Physics, Chemistry and Mathematics*, 2(1), 6-12.
Available at: www.noveltyjournals.com

Konarasinghe, W.G.S., Abeynayake, N.R. (2015). Fourier Transformation on Model Fitting for Sri Lankan Share Market Returns. *Global Journal for Research Analysis*, 4(1), 159-161. Available at: http://theglobaljournals.com/

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2015). Covariance Structure of Sri Lankan Share Market Returns. *International Journal of Novel Research in Interdisciplinary Studies*, 2(1), 13-22. Available at: www.noveltyjournals.com

Konarasinghe, W.G.S. (2014). Review of Statistical Modeling in Technical Analysis of Financial markets. *Journal of Management, South Eastern University of Sri Lanka*, X(1), 42-50.

http://ir.lib.seu.ac.lk/handle/123456789/1413

Konarasinghe, W.G.S. & Pathirawasam, C. (2013). Modeling Stock Returns and Trading Volume of Colombo Stock Exchange. *Sri Lankan Journal of Management*, 18, (3&4), 167-224

Available at: http://www.sljm.pim.lk/

Conference Proceedings

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2016). Circular Methods on Forecasting Risk & Return of Hotel & Travel Sector of Sri Lankan Share Market. *Proceedings of International Conference on Business, Economics, Social Sciences & Humanities, Singapore*, 197, (14), 18.

Available at: http://academicfora.com/

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2015). Spectral Analysis on Forecasting Sri Lankan Share Market Returns. *Proceedings of Academics World 7th International Conference,* Bangkok, Thailand, 13-18.

Available at: http://worldresearchlibrary.org/

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2015). Fourier analysis on Modeling Sector Returns of Sri Lankan Share Market. *Conference Proceedings of International Conference of Multidisciplinary Approaches- 2015*, Faculty of Graduate Studies, University of Sri Jayewardenepura, Sri Lanka, 135.

Konarasinghe, W.G.S., Abeynayake, N.R., Gunaratne, L.H.P. (2015). Comparison of Time Domain and Frequency Domain Analysis in Forecasting Sri Lankan Share Market Returns. *Proceedings of ISER 2nd International Conference*, Singapore, 1-6.

Konarasinghe, W.G.S., Abeynayake, N.R. (2015). Fourier Analysis on Modeling Stock Returns of Individual Companies in Bank Finance & Insurance Sector of Sri Lanka, Share Market. *Conference Proceedings of 2nd Ruhuna International Science and Technology Conference*, Faculty of Science, University of Ruhuna, Sri Lanka, 103.

Available at http://www.sci.ruh.ac.lk/

Konarasinghe, W.G.S., Abeynayake, N.R. (2014). Time Series Patterns of Sri Lankan Stock Returns. *Proceedings of Doctoral Consortium, 11th International Conference in Business & Management*, University of Sri Jayewardenepura, Sri Lanka, 78-95.

Konarasinghe, W.G.S., Abeynayake, N.R. (2014). ARIMA Models on Forecasting Stock Returns. *Annual Academic Sessions*, The Open University of Sri Lanka, 280.

Konarasinghe, W.G.S., Abeynayake, N.R. (2014).Review of Statistical Modeling in Technical Analysis of Financial Markets. *Conference Proceedings of Wayamba*

International Conference-2014, Wayamba University of Srilanka, Kuliyapitiya, Sri Lanka, 300.

Konarasinghe, W.G.S., Abeynayake, N.R. (2014).Covariance Structure of Share Market Returns. *Conference Proceedings of International Conference of Multidisciplinary Approaches- 2014*, Faculty of Graduate Studies, University of Sri Jayewardenepura, Sri Lanka, 93.

Konarasinghe, W.G.S., Abeynayake, N.R. (2014).Modeling Stock Returns of Individual Companies of Colombo Stock Exchange. *Conference Proceedings of the 1st International Forum for Mathematical Modeling- 2014*, Department of Mathematics, University of Colombo, Sri Lanka, 111.

REFERENCES

Anderson, T., W. (1971). The Statistical Analysis of Time Series. *Willey Classic Library*, John Wiley & Sons, New York.

Attwood, G., Clegg, A., Dyer, G., & Dyer, J. (2008). Edexcel AS and A Level Modular Mathematics: Statistics 1. *Pearson Education Limite,* England & Wales.

Attwood, G., Cope, L., Moran, B., Pateman, L., Pledger, K., Staley, G., & Wilkins, G. (2008). Edexcel AS and A Level Modular Mathematics: Further Pure mathematics 1. *Pearson Education Limited,* England & Wales.

Ayodele A. A., Aderemi O. A., Charles K. A. (2014-a). Stock Price Predictions Using ARIMA Models. *UKSim-AMSS 16th International Conference on Computer Modeling and Simulation proceedings*, 105-111.

Ayodele A. A., Aderemi O. A., Charles K. A.,(2014-b). Comparison of ARIMA and Artificial Neural Networks Models for Stock Price Prediction. *Journal of Applied Mathematics*, Volume 2014, 1-6.

Retrieved from: http://dx.doi.org/10.1155/2014/614342

Banz, R. (1981). The Relationship between Returns and Market Value of Common Stocks. Journal of Financial Economics, 9, 3–18.

Basu, S. (1983). The Relationship between Earning Yields, Market Value, and Return for NYSE Common Stock: Further Evidence. *Journal of Financial Economics*, 12, 129–156.

Bhandari, L. C. (1988). Debt/ Equity Ratio and Expected Common Returns: Empirical Evidence. *Journal of Finance*, XLIII(2).

Black, F., Jensen, M., & Scholes, S. (1972). The Capital Asset Pricing Model: Some Empirical Tests. *Social Science Research Network (SSRN).*
Retrieved from: http://ssrn.com/abstract=908569

Chan,K., L., Hamao, Y., Lakonishok, J. (1991), Fundamentals and stock returns in Japan, *Journal of Finance* 46, 1739-1789.

Chordia, T., Swaminathan, B. (2000). Trading Volume and Cross Auto-correlations in Stock Returns. *Journal of Finance*, LV (2).

Clark, P.K.A. (1973). Subordinated Stochastic Process Model with Finite Variance for Speculative Prices. *Econometrica*, 41 (1), 135-155.

Crouch, R.L. (1970). The Volume of Transactions and Price Changes on the New York Stock Exchange. *Financial Analyst Journal*.

Ciner, C. (2003). Dynamic linkages between trading volume and price movements: Evidence for small firm stocks. *Journal of Entrepreneurial Finance*. 8(1), 1-15.

Fama, E.F., Kenneth, R. F. (1992). The Cross –Section of Expected Stock Returns. *Journal of Finance,* XLVII (2), 427-465.

Fama, E. F., James, M. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy*, 81, 607-636.

Fama, E.F., Kenneth, R. F. (1999). Characteristics, Covariance's, and Average Returns: 1929-1997. Social Science Research Network (SSRN).

Retrieved from: http://ssrn.com/

Fama, E.F., Macbeth, J., D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy,* 81(3), 607-636

Emenike, K.O. (2010). Forecasting Nigerian Stock Exchange Returns: Evidence from autoregressive Integrated Moving Average (ARIMA) Model. *Department of Banking and Finance, University of Nigeria, Enugu Campus*, Enugu State, Nigeria, 2010.

Retrieved from http://ssrn.com/abstract=1633006/

Granger, C.W.J., Morgenstem, O. (1963). Spectral Analysis of New York Stock Market Prices. *Willey Online Library, International Review of Social Sciences, KYKLOS,* 16(1), 1-27.

Granger, C.W.J., Hatanaka, M. (1964). Spectral Analysis of Economic Time Series. *This Weeks Citation Classic*, Princeton University Press, Princeton, New Jersey, 299.

Gong- Meng, C., Michael, F., Oliver, M., R. (2001). The Dynamic Relation Between Stocks Returns, Trading Volume, and Volatility. *The Financial Review*. 38, 153-174.

Guillrmo, L., Roni, M., Gideon, S., Jiang, W. (2002). Dynamic Volume- Return of Individual Stocks. *The Society of financial studies*. 15(4),1005-1047.

Gujarati, D., N., Porter, D., C., Gunasekar, S. (2009). Basic Econometrics. *McGraw Hill Education(India) Private Limited*, New Delhi.

Habib, N.M. (2011).Trade Volume and Returns In Emerging Stock Markets An Empirical Study: The Egyptian Market. *International Journal of Humanities and Social Science,* 1(19).

Heimstra, C., Johathan, D.J. (1994). Testing for Linear and Nonlinear Granger Causality in the Stock Price- Volume Relation. *Journal of Finance*, XLIX (5).

Hooker, S., Jennings, M., Littlewood, J., B., Moran, B., Pateman L. (2009). Edexcel AS and A-Level Modular Mathematics: Mechanics 4, Pearson Education Limited, England & Wales.

Javed, A.Y. (1993). Alternative Capital Asset Pricing Models: A Review of Theory and Evidence.

Jianping, M., Olesya V.G., Lubomir.P. L. (2002). Measuring Private Information Trading in Emerging Markets. New York University.

Kamath, R.R. (2007). Investigating Causal Relations between Price Changes and Trading Volume Changes in the Turkish Market. *ASBBS E-Journal*, 3(1).

Konarasinghe, K.M.U.B. (2015). Trend Analysis of Direct Employment in Tourism Industry of Sri Lanka. *Conference Proceedings of the 4th International Conference of the Sri Lankan Forum of University Economists.* Department of Business Economics, Facultyof Management Studies and Commerce, University of Sri Jayawardanapura, Sri Lanka, 31.

Konarasinghe, W.G.S., Pathirawasam, C. (2013-a).The Dynamic Relation between Stock Returns and Trading Volume of Colombo Stock Exchange. *Proceedings of the 2nd International Research Conference on Humanities and Social Sciences 2013 ,* Faculty of Humanities and Social Sciences University of Sri Jayawardanapura, Sri Lanka,119.

Konarasinghe, W.G.S., Pathirawasam, C. (2013-b). Modeling Stock Returns and Trading Volume of Colombo Stock Exchange. *Sri Lankan Journal of Management*, 18, (3&4), 167-224

Lintner, J. (1965). The Valuation of Risk Assets and Selection of Risky Investments in Stock Portfolio and Capital Budgets. *Review of Economics and Statistics,* 47, 13–47.

Malabika, D., Srinivasan K.K., Devanadhen, K. (2008). The Empirical Relationship between Stocks Returns, Trading Volume and Volatility: Evidence from Select Asia-Pacific Stock Market. *European Journal of Economics, Finance and Administrative Sciences*, 13, 20-25.

Marwan, D. (2012). Testing the Contemporaneous and Causal Relationship between Trading Volume and Return in the Palestine Exchange. *International Journal of Economics and Finance*. 4 (4), 182-192.

Markovitz, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1) 77-91.

MATLAB & Simulink – MathWorks, Available at: http://in.mathworks.com/

McCullagh, P. (2002). What is a Statistical Model? *The Annals of Statistics*, 30(5), 1225–1310.

Mossin, J. (1966). Equilibrium in a Capital Asset Market. *ECONOMETRICA*, 34(4), 768-783.

Nimal, P.D. (1997). Relationship between Stock Returns and Selected Fundamental Variables; Evidence from Sri Lanka. *Sri Lankan Journal of Management*, 2(3), 1-19.

Naliniprava, T. (2011). The Relation between Price Changes and Trading Volume: A Study in Indian Stock Market. *Interdisciplinary Journal of Research in Business,* 1 (7).81-95.

Osborne, M.F.M. (1959). Brownian motion in the Stock Market. *U.S. Naval Research Laboratory*, Washington.

Ong Sheue, L., Ho Chong, M. (2011). Testing For Linear and Nonlinear Granger Causality in the Stock Return and Stock Trading Volume Relation: Malaysia and Singapore Cases. *Labuan Bulletin of International Business & Finance,* 9.

Pande, I., M. (2005). Financial Management, 9th Edition. Indian Institute of Management, Ahamedabad.

Philippe, M. (2008). Analysis of Financial Time Series Using Fourier and Wavelet Methods. *Social Science Research Network (SSRN).*
Retrieved from: http://ssrn.com/abstract=1289420

Prapanna, M., Labani S., Saptarsi G. (2014). Study of Effectiveness of Time Series Modelling (ARIMA) in Forecasting Stock Prices. *International Journal of Computer Science, Engineering and Applications* (IJCSEA), .4(2), 13-29.

Rathnayaka, R.M.K.T., Seneviratna, D.M.K.N., Nagahawatta, S.C. (2014). Empirical Investigation of Stock Market Behaviour in the Colombo Stock Exchange. *Proceedings of the 3rd International Conference on Management and Economics,Faculty of Management and Finance, University of Ruhuna*. 209-216.

Rosangela, B., Ivette L., Lilian M., Rodrigo, L. (2010).A comparative analysis of Eurofuzzy, ANN and ARIMA models for Brazilian stock index forecasting. Department of Economic Theory, Institute of Economics, University of Campinas, Brazil.

Rozenberg, B., Reid, K., Lanstein, R. (1985). Persuasive Evidence of Market Inefficiency. *Journal of Portfolio Management*, 11, 9-17.

Samarakoon, M. P. (1997). The Cross Section of Expected Stock Returns of Sri Lanka". Sri Lankan Journal of Management, 2(3), 19-35.

Samarakoon, L. (2010).Equity Securities, Theory & Practice. The Securities and Exchange Commission of Sri Lanka.

Saatcioglu, K., & Starks, L.T. (1998). The Stock Price-Volume Relationship in Emerging Stock Markets: The Case of Latin America. *International Journal of Forecasting*.14 (2), 215-225.

Sarika, M., Balwinder, S. (2009). The Empirical Investigation of Relationship between Return, Volume and Volatility Dynamics in Indian Stock Market. *Eurasian Journal of Business and Economics 2009*. 2 (4),113-137.

Sharpe, W. F. (1964).Capital Asset Pricing Theory of Market Equilibrium under Conditions if Risk. *Journal of Finance* 19, 425–442.

Sims, C. (1980), "Macroeconomics and Reality", *Econometrica,* 48, 1-48.

Stattman, D. (1980), Book Values and Stock Returns, *The Chicgo MBA: Journal of Selected Papers*, 4, 25-45.

Stephen, A. D. (1998). Forecasting Principles and Applications. Irwin / McGraw-Hill, USA.

Sharpe, W. F. (1964). Capital Asset Pricing Theory of Market Equilibrium under Conditions if Risk. *Journal of Finance*, 19, 425–442.

Tabin, J. (1958). Liquidity Preferences as Behaviour Towards Risk. *Review of Economic Studies*, 25, 65–86,

Timothy, S.M. (1992). Portfolio returns autocorrelation. *Journal of Financial Economics*, 34,307-344.

Timothy, J.B. (1994). The Empirical Relationship between Trading Volume, Returns and Volatility. Asia-Pacific Finance Association Conference.

Treynor, J., L. (1961).Towards a Theory of Market Value of Risky Assets, (Unpublished Manuscript).

Wikipedia, the free encyclopedia (2016). Colombo Stock Exchange.

Retrieved from:https://en.wikipedia.org/wiki/Colombo_Stock_Exchange.

Wen-Hsiu, K., Hsinan, H., Chwan-Yi, C. (2004). Trading Volume and Cross-Autocorrelations of Stock Returns in Emerging Markets: Evidence from the Taiwan Stock Market. *Review of Pacific Basin Financial Markets and Policies*.7 (4), 509–524 .

Xiangmei, F., Nicolaas, G., & Yanrui, W. (2003). The Stock Return-volume Relation and Policy Effects: The Case of the Chinese Energy Sector. *Proceedings of the 15th Annual Conference of the Association for Chinese Economics Studies Australia (ACESA)*.

BIBLIOGRAPHY

Basu, S. (1977). Investment Performance of Common Stocks in Relation to Their Price Earnings Ratios: A Test of Efficient Market Hypothesis. *Journal of Finance*.

Box, G.P.E. and Jenkins, G.M. (1976). Time Series Analysis: Forecasting and Control, Holden Day: San Francisco.

Chan, L., Yasushi, H., Josef, L. (1991). Fundamentals and stock returns in Japan. *Journal of Finance,* 46, 1739-1789.

Chia-Chang, C., Chung-Ming K., & Hsin-Yi. L. (2009). Causality in Quantiles and Dynamic Stock Return-Volume Relations. National Taipei College of Business, National Taiwan University, National Chengchi University.

CSE. (2014). Annual Report. Colombo Stock Exchange.

Das, P., K. (1995). Non-linear Statistical Models for Studying Acreage, Production and Productivity of Wheat in India. Indian Agricultural Research Institute, New Dhlhi, 110 012.

Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50(4), 987-1007.

Fama, E.F. (1965). "The Behavior of Stock Market Prices." *Journal of Business*, 38, 34-105.

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *Journal of Finance*, 25, 383 – 417.

Gopikrishnan, P., Plerou, V., Gabaix, X., Eugene, S., H. (2000). Statistical properties of share volume traded in financial markets. *Physical Review,* 62(4).

Kalman, J. C., Gabriel, A. H., Steven, F. M., Robert, A. S. David, K. W. (1980) Implications of Microstructure Theory for Empirical Research on Stock Price Behavior. *Journal of Finance.* Xxxv (2).

Kamath, R.R. (2007). Investigating Causal Relations between Price Changes and Trading Volume Changes in the Turkish Market. *ASBBS E-Journal,* 3 (1).

Karpoff, J.M. (1987). The Relation between Price Changes and Trading Volume: A Survey. *Analysis Journal of Financial and Quantitative.* 22 (1).

Levy, H., Ran, D. (2004). Asset Return Distributions and the Investment Horizon. *The Journal of Portfolio Management.*

Le Quang, T., Mustafa, M. (2009). The Relationship between Trading Volume, Stock Index Returns and Volatility: Empirical Evidence in Nordic Countries. Lund University School of Economics and Management.

Pathirawasam, C. (2009). The Relationship between Trading Volume and Stock Returns. *Journal of Competitiveness,* 3.

Rahman, S., Hossain, M.F. (2006).Weak Form Efficiency: Testimony of Dhaka Stock Exchange. *Journal of Business Research*, 8, 1-12.

Schumacher, C. R. (1997). Probability Distribution of Stock Market Returns. *Western Michigan University*.

Uma, D. B., Sundar D., Ali, P. (2013). An effective Time Series Analysis for Stock Trend Prediction using ARIMA models for Nifty Midcap-50. *International Journal of Data Mining and Knowledge Management Process*, 3(1).

Wolfgang H., Marlene M., Stefan S., Axel W. (2005). Non parametric and Semi parametric Models. Retrieved from http://www.getiezewdie.com/.

Wong, W.,K., Manzur, M., Chew, B.K. (2002). How Rewarding Is Technical Analysis? Evidence from Singapore Stock Market. National University of Singapore, Department of Economics, Working Paper, 0216.

Appendix 1: Computer Programs used for the Data Analysis

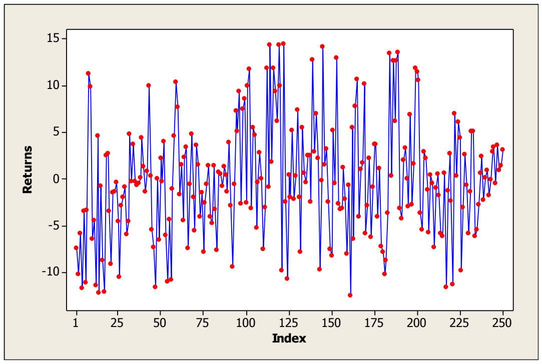
1.1 MATLAB Program for the Outlier Detection and Adjustment

```
x=double(dataset(....txt'));

m = prctile(x,50)

Qone=prctile(x,25)

 Qthree=prctile(x,75)

IQR=Qthree-Qone

LB=Qone-1.5*IQR

UB=Qthree+1.5*IQR

for i=4:length(x)

   if x(i) > UB || x(i) < LB;

       y(i)=((x(i-1)+x(i-2)+x(i-3))/3);

 else

       y(i)=x(i);

   end

end

y'
```

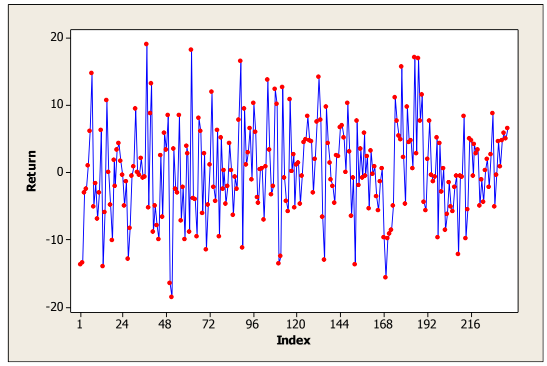
1.2 MATLAB Program for fitting Circular Model

```
y=double(dataset(….txt'));  yc=y-mean(y);

line([1,],[mean(y),mean(y)]);  cs=sy(1);

once_changed=0;  No_of_waves=0;

for i=2:length(sy)

    if cs~=sy(i)

       cs=sy(i);

        if ~once_changed

            once_changed=1;

        else  No_of_waves=No_of_waves+1;

            once_changed=0;

        end     end   end

No_of_waves; f=No_of_waves

N=length(y); om=(2*pi*f/N); t=[1:]'; Ary1=[];

for k=1:6 Ary1(:,k)=sin(k*om*t);

end

Ary1; Ary2=[];

for k=1:6 Ary2(:,k)=cos(k*om*t);

end Ary2;

X=[Ary1,Ary2]; mdl = GeneralizedLinearModel.fit(X,y)
```

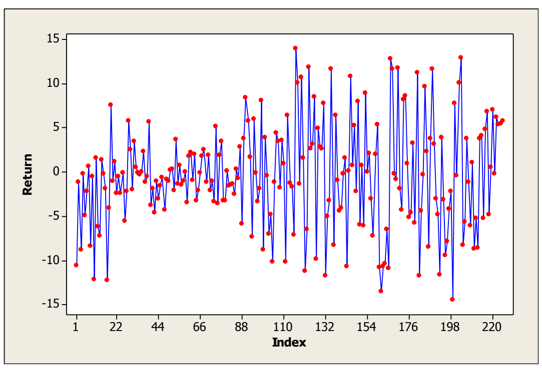
APPENDIX 2: TIME SERIES PLOTS OF RETURNS



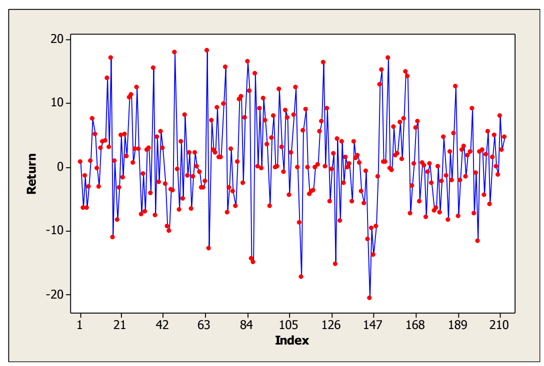
2.1 Time Series Plot-Sector H&T



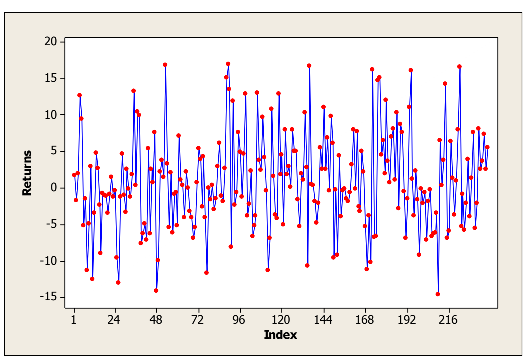
2.2 Time Series Plot-Sector BFI



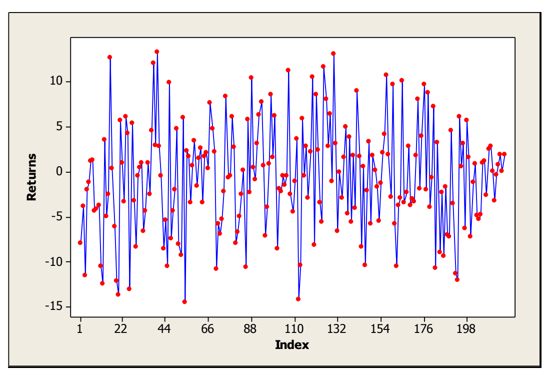
2.3 Time Series Plot-Sector L&P



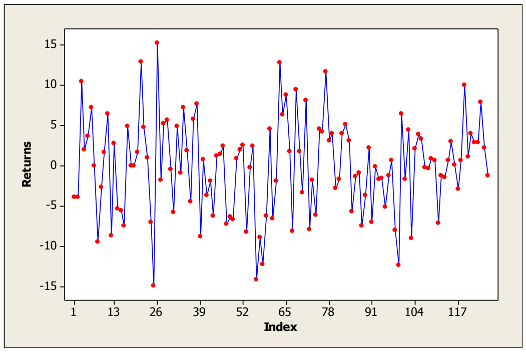
2.4 Time Series Plot-Sector DIV



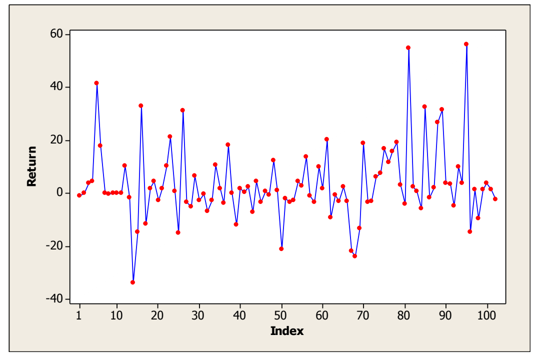
2.5 Time Series Plot-Sector MFG



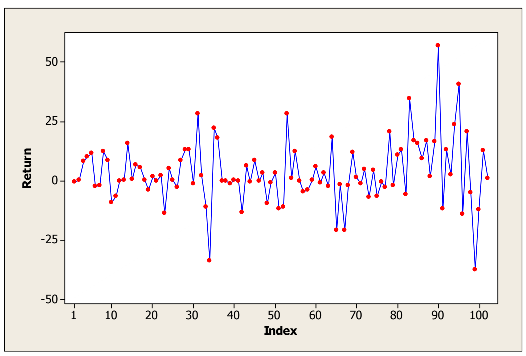
2.6 Time Series Plot-Sector MTR



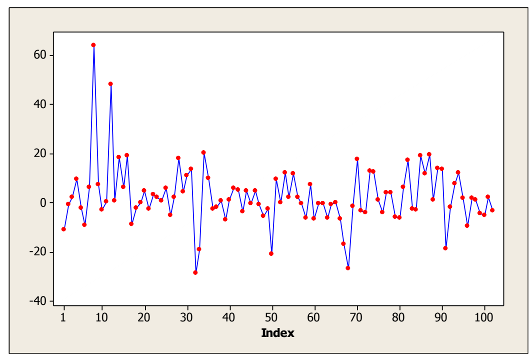
2.7 Time Series Plot-Sector TLE



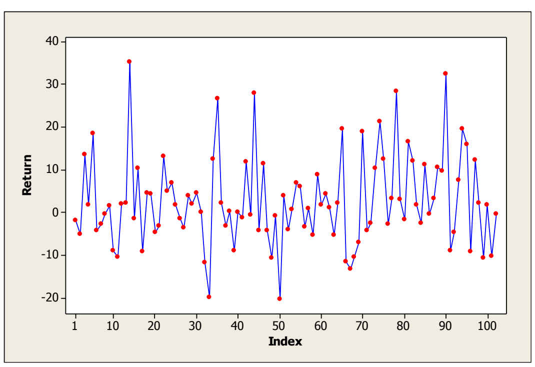
2.8 Time Series Plot-Sector OIL



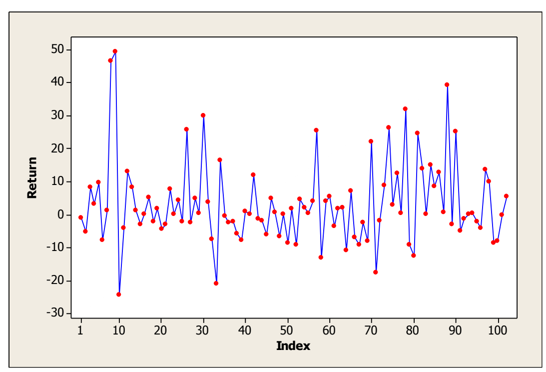
2.9 Time Series Plot-Sector S&S



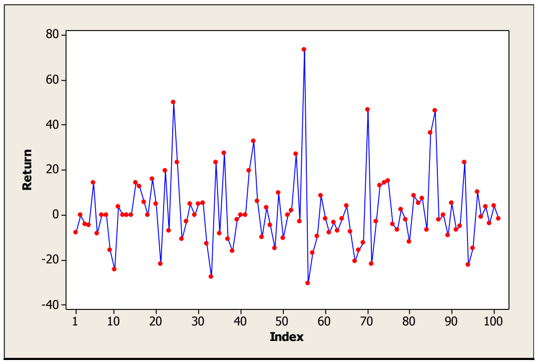
2.10 Time Series Plot-Sector F&T



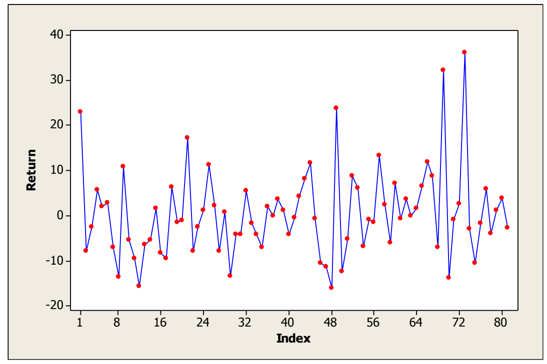
2.11 Time Series Plot-Sector C&P



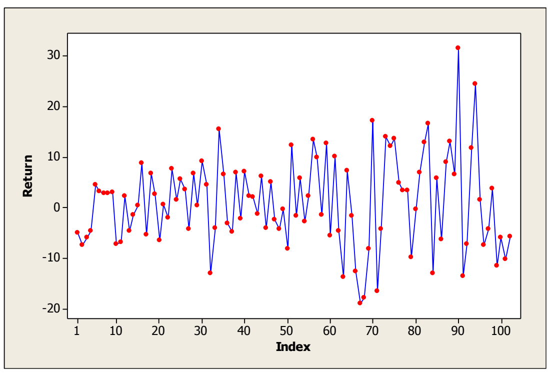
2.12 Time Series Plot-Sector SRV



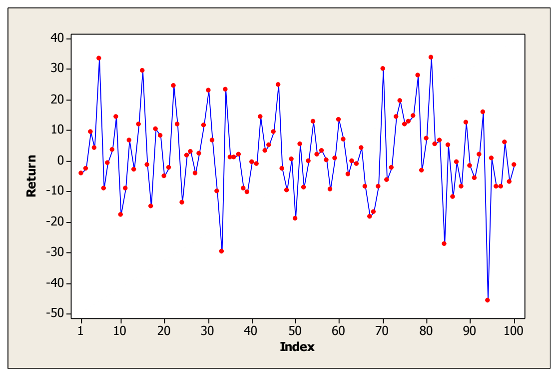
2.13 Time Series Plot-Sector IT



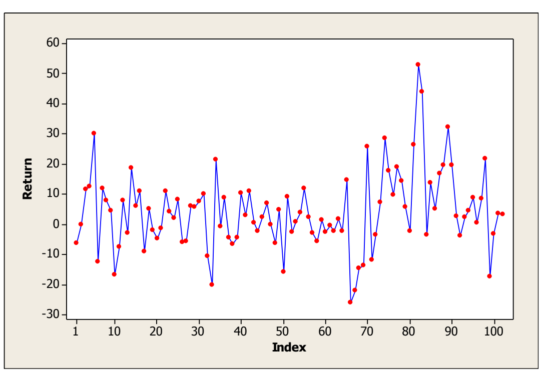
2.14 Time Series Plot-Sector P&E



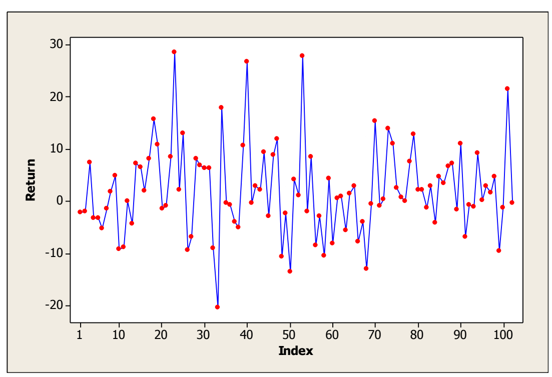
2.15 Time Series Plot-Sector PLT



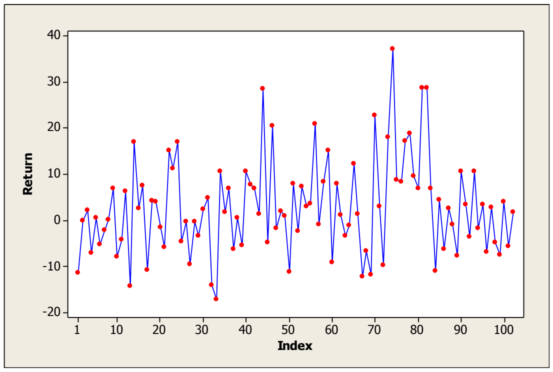
2.16 Time Series Plot-Sector INV



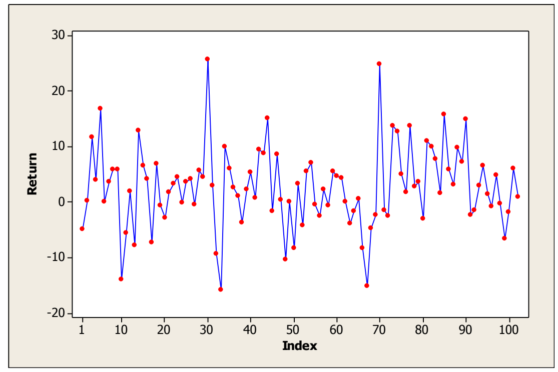
2.17 Time Series Plot-Sector TRD



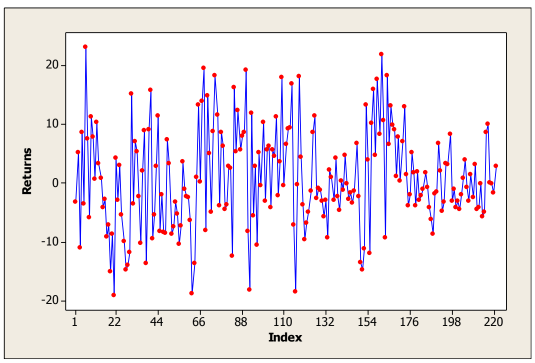
2.18 Time Series Plot-Sector HLT



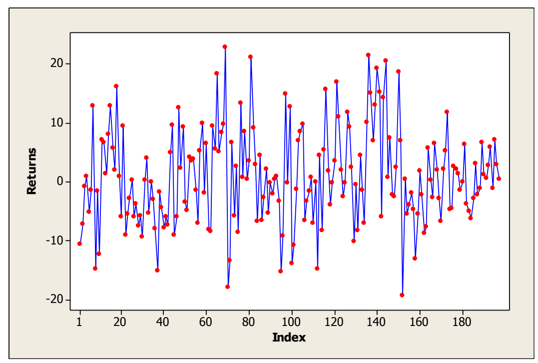
2.19 Time Series Plot-Sector C&E



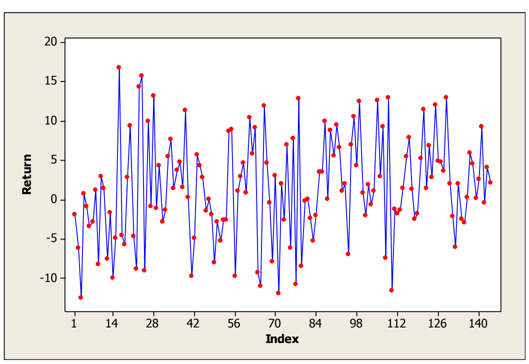
2.20 Time Series Plot-Sector BFT



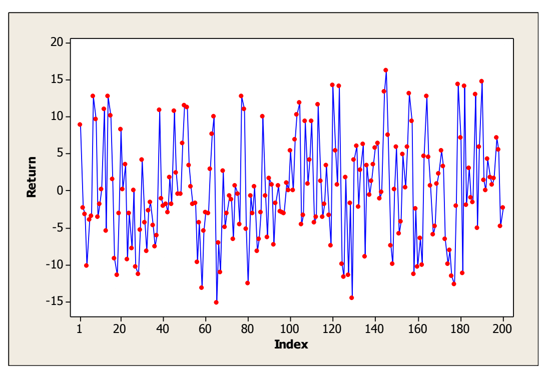
2.21 Time Series Plot-AHOT



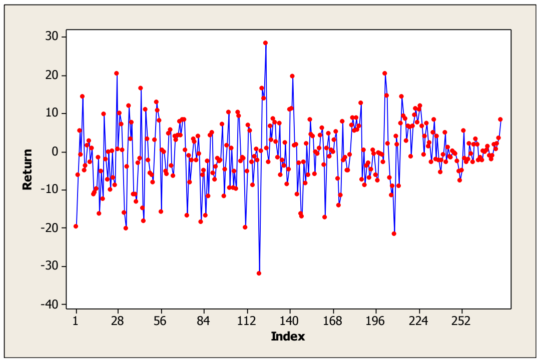
2.22 Time Series Plot-AHUN



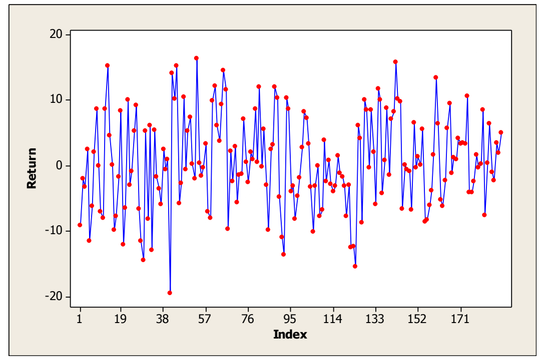
2.23 Time Series Plot-BREW



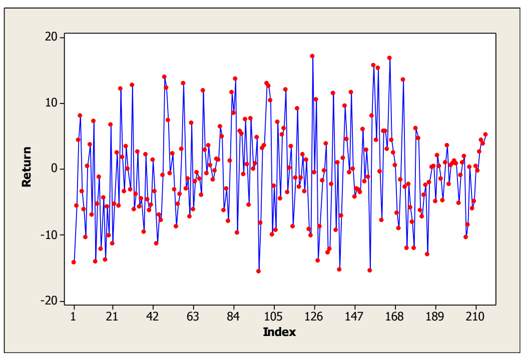
2.24 Time Series Plot-DISTIL



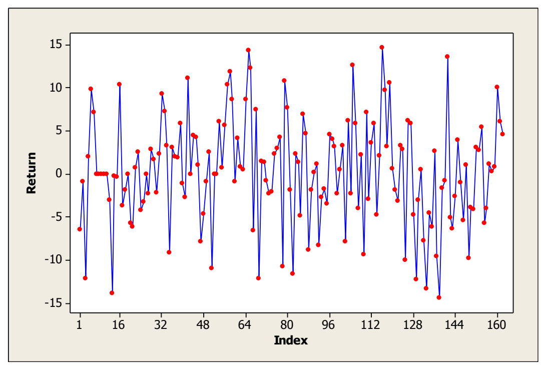
2.25 Time Series Plot- HAYL



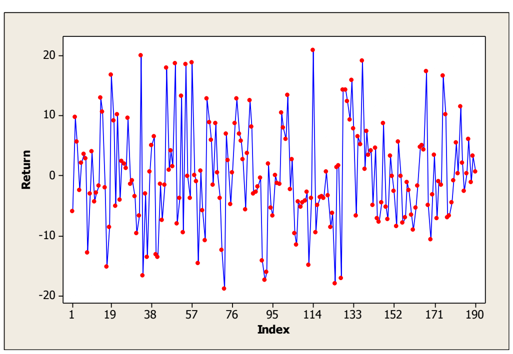
2.26 Time Series Plot-JKH



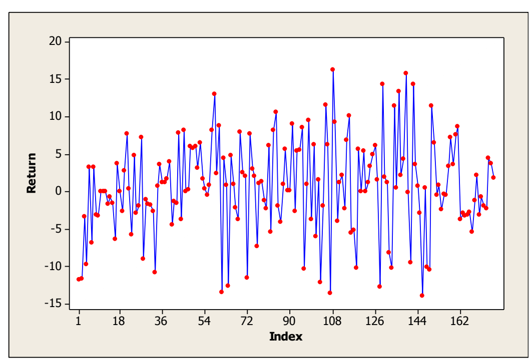
2.27 Time Series Plot-EDEN



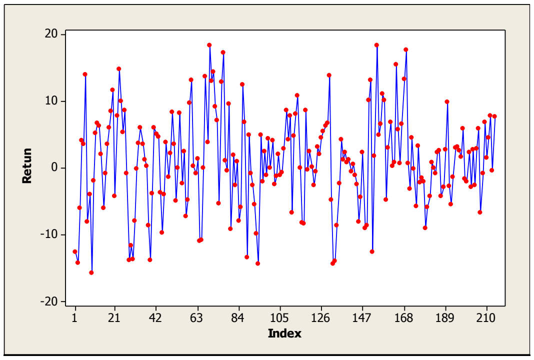
2.28 Time Series Plot-ABAN



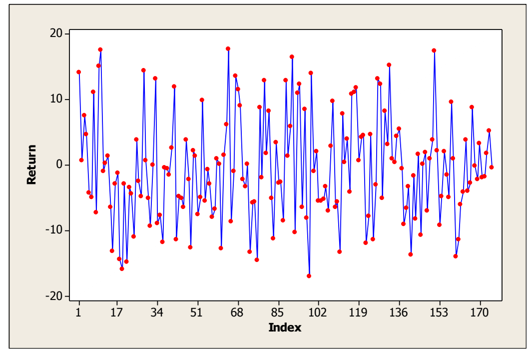
2.29 Time Series Plot-KELANI



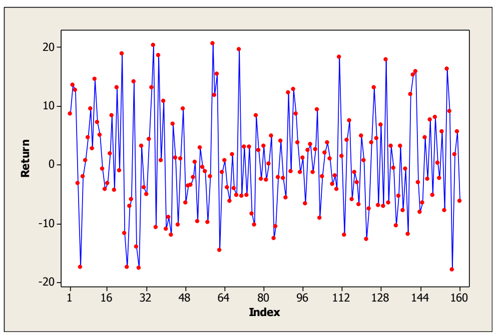
2.30 Time Series Plot-ALLI



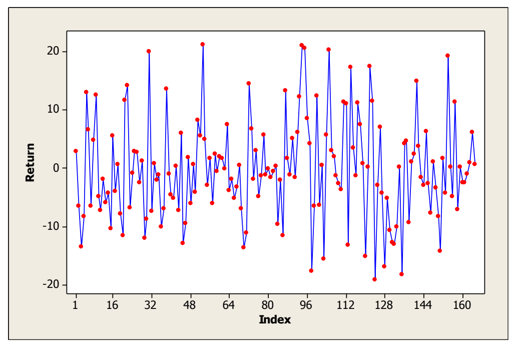
2.31 Time Series Plot-COMB



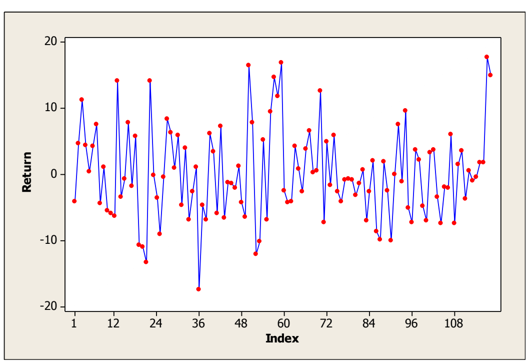
2.32 Time Series Plot-AGAL



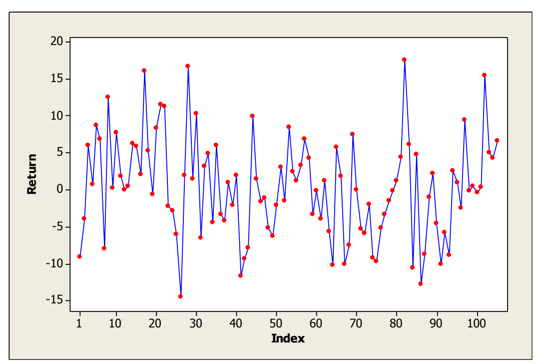
2.33 Time Series Plot-BOGA



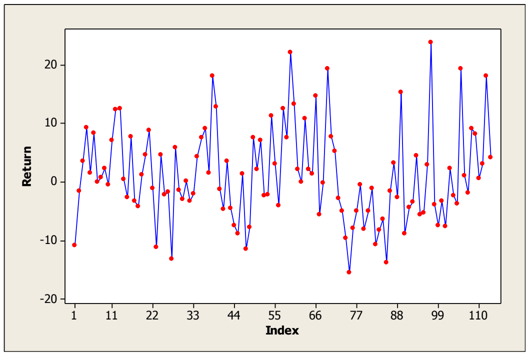
2.34 Time Series Plot-BALA



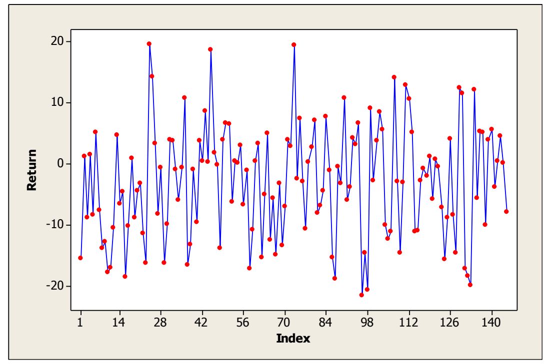
2.35 Time Series Plot-HUNA



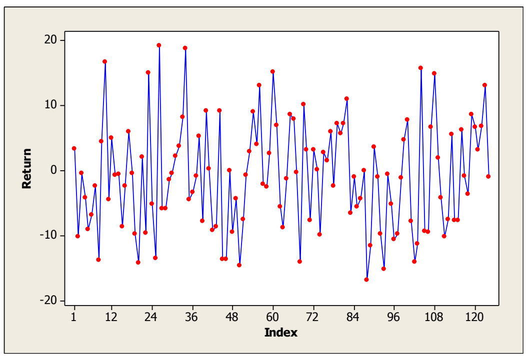
2.36 Time Series Plot-PALM



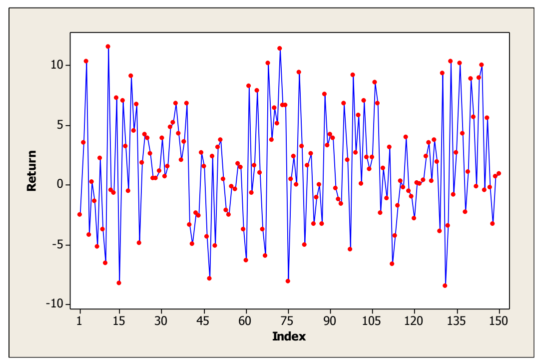
2.37 Time Series Plot-ROCEL



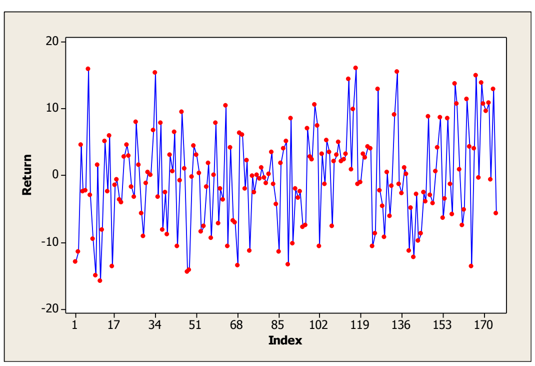
2.38 Time Series Plot-BLUE



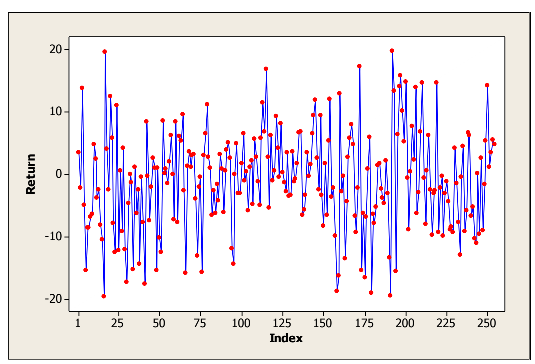
2.39 Time Series Plot-BOGAL



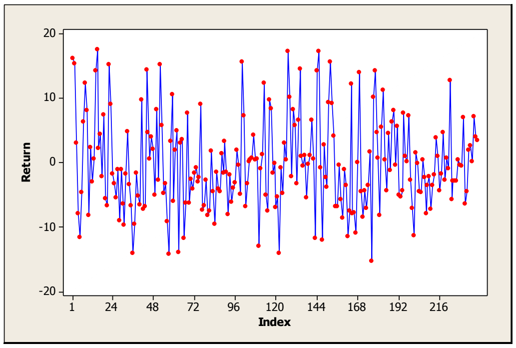
2.40 Time Series Plot-NEST



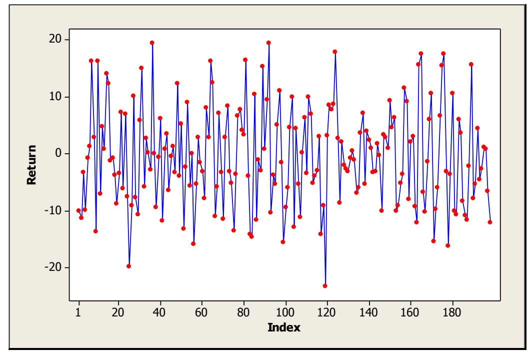
2.41 Time Series Plot-RICHA



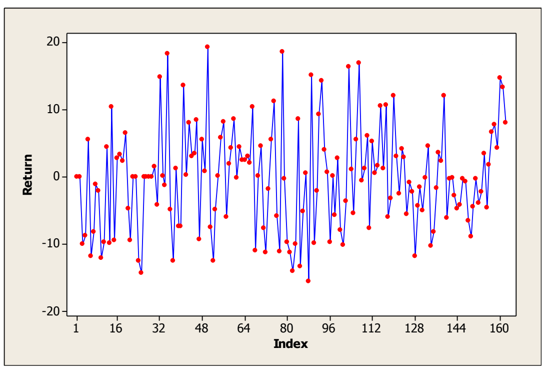
2.42 Time Series Plot-GHLL



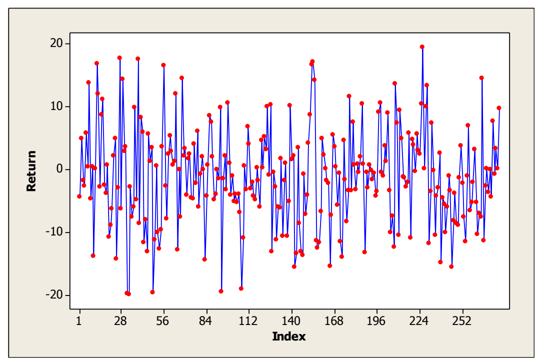
2.43 Time Series Plot-ACL



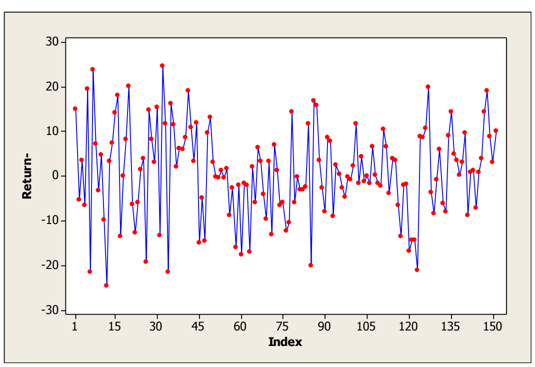
2.44 Time Series Plot-ACME



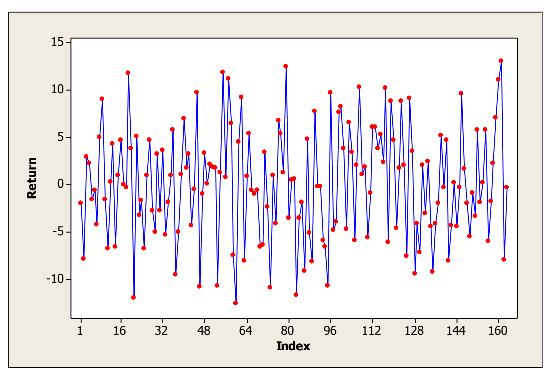
2.45 Time Series Plot-LFIN



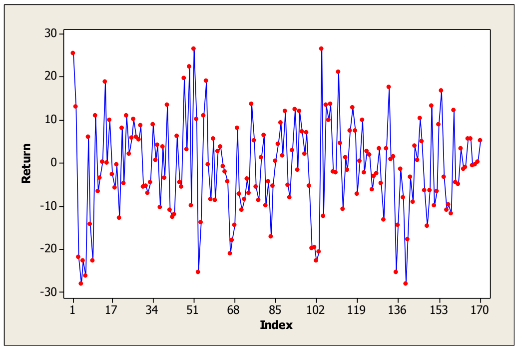
2.46 Time Series Plot-CLAND



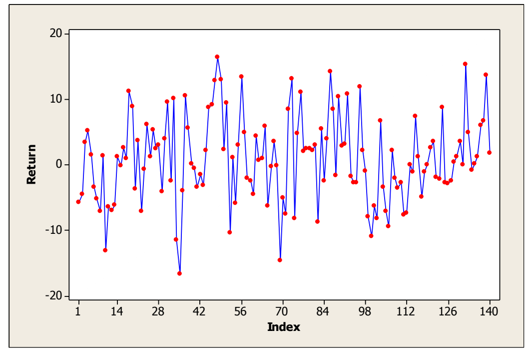
2.47 Time Series Plot-KELSEY



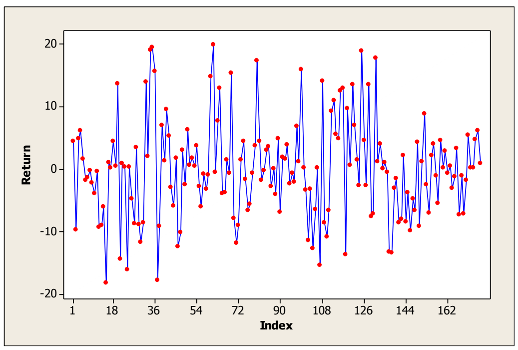
2.48 Time Series Plot-PDL



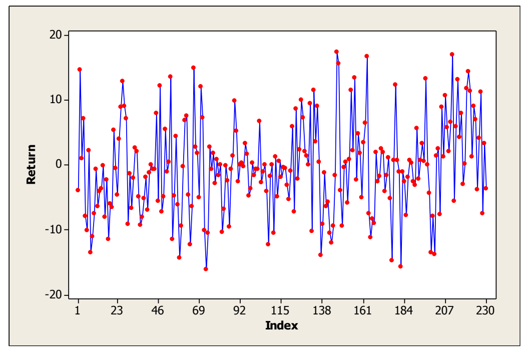
2.49 Time Series Plot-EAST



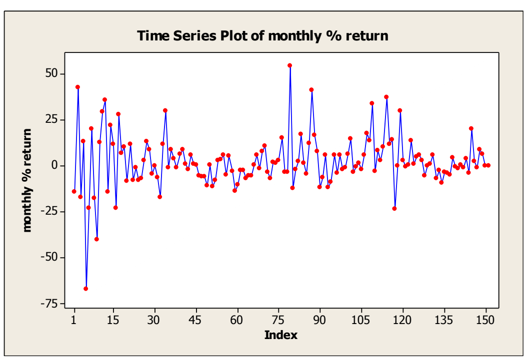
2.50 Time Series Plot-HASU



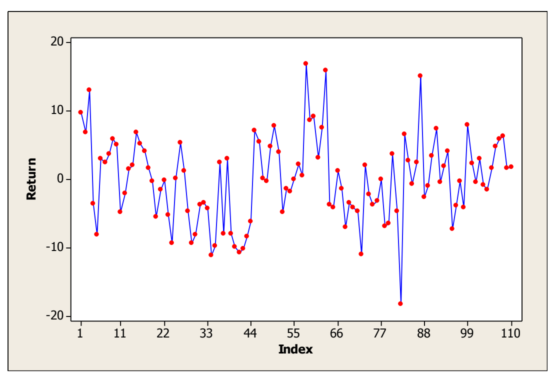
2.51 Time Series Plot-CERA



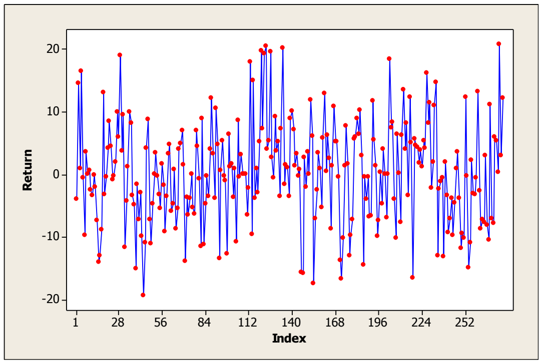
2.52 Time Series Plot-DIMO



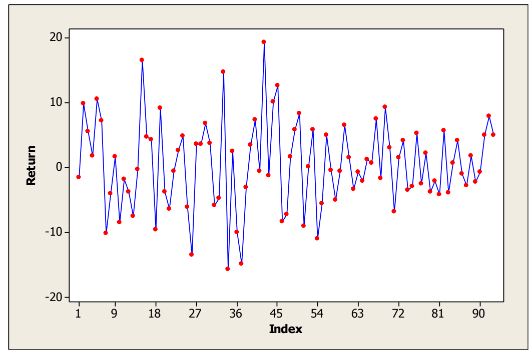
2.53 Time Series Plot-AMW



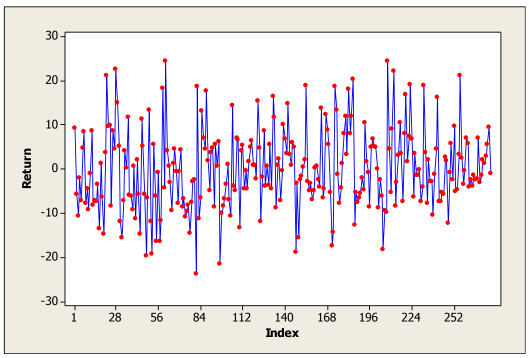
2.54 Time Series Plot-DIAL



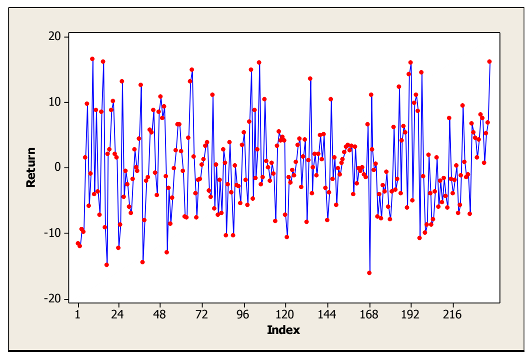
2.55 Time Series Plot-CIC



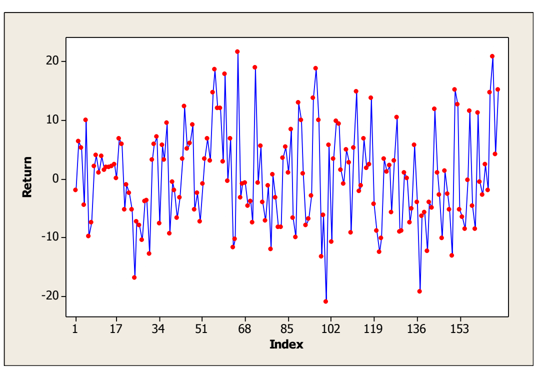
2.56 Time Series Plot-SIGI



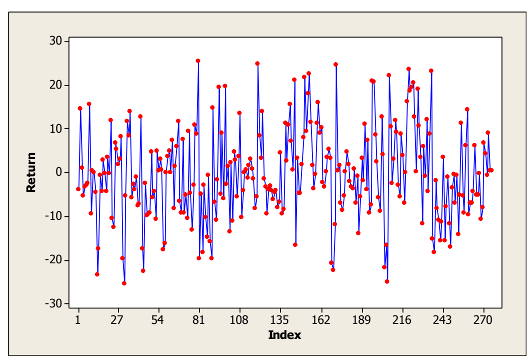
2.57 Time Series Plot-LMF



2.58 Time Series Plot-TOKYO



2.59 Time Series Plot-EQIT



2.60 Time Series Plot-LANK

APPENDIX 3: ADFT AND ENGLE'S ARCH TEST RESULTS

3.1 Augmented Dickey Fuller Test (ADFT) Results

Sector	**Company**	**ADFT results**				
		h	**P Value**	**Test Statistic**	**Critical**	**Comments**
H&T	PEGA	1	1.0000e-03	-10.7214	-1.9425	Stationary
	GHLL	1	1.0000e-03	-12.3851	-1.9421	Stationary
	TRANS	1	1.0000e-03	-13.4796	-1.9423	Stationary
	HUNA	1	1.0000e-03	-8.2448	-1.9446	Stationary
	PALM	1	1.0000e-03	-6.6417	-1.9446	Stationary
	SIGI	1	1.0000e-03	-7.5797	-1.9459	Stationary
	AHOT	1	1.0000e-03	-10.9495	-1.9425	Stationary
	AHUN	1	1.0000e-03	-9.5730	-1.9425	Stationary
	ABAN	1	1.0000e-03	-9.6042	-1.9439	Stationary
MFG	ACL	1	1.0000e-03	-11.6049	-1.9423	Stationary
	ACME	1	1.0000e-03	-11.3291	-1.9425	Stationary
	KELANI	1	1.0000e-03	-10.6370	-1.9425	Stationary
	WALL	1	1.0000e-03	-6.4314	-1.9446	Stationary
	ROCEL	1	1.0000e-03	-6.7616	-1.9444	Stationary
	BLUE	1	1.0000e-03	-8.4001	-1.9444	Stationary
	BOGALA	1	1.0000e-03	-8.8674	-1.9445	Stationary
	CERA	1	1.0000e-03	-10.0675	-1.9429	Stationary
BFI	ALLI	1	1.0000e-03	-13.4584	-1.9424	Stationary
	ASIA	1	1.0000e-03	-10.3273	-1.9424	Stationary
	COMB	1	1.0000e-03	-10.6274	-1.9423	Stationary
PLT	AGAL	1	1.0000e-03	-10.0942	-1.9425	Stationary
	BALA	1	1.0000e-03	-10.0668	-1.9433	Stationary
	BOGA	1	1.0000e-03	-11.1215	-1.9424	Stationary
BFT	BREW	1	1.0000e-03	-9.9056	-1.9433	Stationary
	DISTIL	1	1.0000e-03	-11.2835	-1.9423	Stationary
	NESTL	1	1.0000e-03	-10.9515	-1.9425	Stationary
DIV	HAYL	1	1.0000e-03	-10.8940	-1.9421	Stationary
	JKH	1	1.0000e-03	-10.1273	-1.9425	Stationary
L&P	CLAND	1	1.0000e-03	-13.0445	-1.9421	Stationary
	KELSEY	1	1.0000e-03	-11.6504	-1.9441	Stationary

3.2: Engle's ARCH Test Results

Sector	**Company**	**Engle's ARCH Test**		
		h	**P value**	**F stat**
H&T	PEGA	0	0.5382	1.2392
	GHLL	1	9.3220e-04	10.9576
	EDEN	0	0.7550	0.0974
	TAJ	1	0.0308	4.6629
	HUNA	0	0.0954	2.7812
MFG	ABAN	0	0.8372	0.0422
	ACL	0	0.1941	1.6864
	ACME	0	0.5421	0.3716
	LMF	0	0.9595	0.0026
	TOKYO	1	0.0272	4.8802
	WALL	0	0.9299	0.0077
	BOGALA	0	0.7526	0.0993
BFI	ALLI	0	0.4602	0.5454
	ASIA	0	0.0544	3.7003
	COMB	0	0.1391	2.1876
	DFCC	1	0.0022	9.3955
	LFIN	0	0.3628	0.8281
PLT	AGAL	0	0.8560	0.0329
	BALA	0	0.2026	1.6236
	BOGA	1	4.5053e-04	12.3100
BFT	BREW	1	0.0012	10.5268
	DISTIL	0	0.3367	0.9229
	NESTL	0	0.1355	2.2279
DIV	HAYL	0	0.1165	2.4639
	RICHA	0	0.4782	0.5030
L&P	CLAND	0	0.0934	2.8141
	KELSEY	1	0.0213	5.3031
	EAST	1	0.0295	4.7385
C&P	LANK	1	1.7725e-05	18.4194
	CIC	0	0.6632	0.1897

APPENDIX 4: FOURIER TRANSFORMATION

Appendix 4.1: Fourier Transformation on Returns of EDEN

Sinωt	*Sin2ω t*	*Sin3ω t*	*Sin4ω t*	*Sin5ω t*	*Sin6ω t*	*Cosω t*	*Cos2 ωt*	*Cos3 ωt*	*Cos4 ωt*	*Cos5 ωt*	*Cos6 ωt*
0.987 7 0.309 0 - 0.891 0 - 0.587 8 0.707 1 0.809 0 - 0.454 0 - 0.951 1 0.156 4 1.000 0 0.156 4 - 0.951 1 - 0.454 0 0.809 0 0.707 1 - 0.587 8 - 0.891 0 0.309 0 0.987	0.309 0 - 0.587 8 0.809 0 - 0.951 1 1.000 0 - 0.951 1 0.809 0 - 0.587 8 0.309 0 0.000 0 - 0.309 0 0.587 8 - 0.809 0 0.951 1 - 1.000 0 0.951 1 - 0.809 0 0.587 8 -	- 0.891 0 0.809 0 0.156 4 - 0.951 1 0.707 1 0.309 0 - 0.987 7 0.587 8 0.454 0 - 1.000 0 0.454 0 0.587 8 - 0.987 7 0.309 0 0.707 1 - 0.951 1 0.156 4 0.809 0 - 0.891 0	- 0.587 8 - 0.951 1 - 0.951 1 - 0.587 8 0.000 0 0.587 8 0.951 1 0.951 1 0.587 8 0.000 0 - 0.587 8 - 0.951 1 - 0.951 1 - 0.587 8 0.000 0 0.587 8 0.951 1 0.951 1 0.587	0.707 1 1.000 0 0.707 1 0.000 0 - 0.707 1 - 1.000 0 - 0.707 1 0.000 0 0.707 1 1.000 0 0.707 1 0.000 0 - 0.707 1 - 1.000 0 - 0.707 1 0.000 0 0.707 1 1.000 0 0.707 1 0.000	0.809 0 - 0.951 1 0.309 0 0.587 8 - 1.000 0 0.587 8 0.309 0 - 0.951 1 0.809 0 0.000 0 - 0.809 0 0.951 1 - 0.309 0 - 0.587 8 1.000 0 - 0.587 8 - 0.309 0 0.951 1 -	0.156 4 - 0.951 1 - 0.454 0 0.809 0 0.707 1 - 0.587 8 - 0.891 0 0.309 0 0.987 7 0.000 0 - 0.987 7 - 0.309 0 0.891 0 0.587 8 - 0.707 1 - 0.809 0 0.454 0 0.951 1 -	- 0.9511 0.8090 - 0.5878 0.3090 0.0000 - 0.3090 0.5878 - 0.8090 0.9511 - 1.0000 0.9511 - 0.8090 0.5878 - 0.3090 0.0000 0.3090 - 0.5878 0.8090 - 0.9511 1.0000 - 0.9511 0.8090 - 0.5878 0.3090 0.0000 - 0.3090 0.5878	- 0.4540 - 0.5878 0.9877 - 0.3090 - 0.7071 0.9511 - 0.1564 - 0.8090 0.8910 0.0000 - 0.8910 0.8090 0.1564 - 0.9511 0.7071 0.3090 - 0.9877 0.5878 0.4540 - 1.0000 0.4540 0.5878 - 0.9877 0.3090 0.7071 - 0.9511 0.1564	0.809 0.309 -0.309 -0.809 -1.000 -0.809 -0.309 0.309 0.809 1.000 0.809 0.309 -0.309 -0.809 -1.000 -0.809 -0.309 0.309 0.809 1.000 0.809 0.309 -0.309 -0.809 -1.000 -0.809 -0.309	0.7071 0.0000 - 0.7071 - 1.0000 - 0.7071 0.0000 0.7071 1.0000 0.7071 0.0000 - 0.7071 - 1.0000 - 0.7071 0.0000 0.7071 1.0000 0.7071 0.0000 - 0.7071 - 1.0000 - 0.7071 0.0000 0.7071 1.0000 0.7071 0.0000 - 0.7071	- 0.5878 - 0.3090 0.9511 - 0.8090 0.0000 0.8090 - 0.9511 0.3090 0.5878 - 1.0000 0.5878 0.3090 - 0.9511 0.8090 0.0000 - 0.8090 0.9511 - 0.3090 - 0.5878 1.0000 - 0.5878 - 0.3090 0.9511 - 0.8090 0.0000 0.8090 - 0.9511

7 0.000 0 - 0.987 7 - 0.309 0 0.891 0 0.587 8 - 0.707 1 - 0.809 0 0.454 0	0.309 0 0.000 0 0.309 0 - 0.587 8 0.809 0 - 0.951 1 1.000 0 - 0.951 1 0.809 0	0.000 0 0.891 0 - 0.809 0 - 0.156 4 0.951 1 - 0.707 1 - 0.309 0 0.987 7	8 0.000 0 - 0.587 8 - 0.951 1 - 0.951 1 - 0.587 8 0.000 0 0.587 8 0.951 1	0 - 0.707 1 - 1.000 0 - 0.707 1 0.000 0 0.707 1 1.000 0 0.707 1	0.809 0 0.000 0 0.809 0 - 0.951 1 0.309 0 0.587 8 - 1.000 0 0.587 8 0.309 0	0.156 4 - 1.000 0 - 0.156 4 0.951 1 0.454 0 - 0.809 0 - 0.707 1 0.587 8 0.891 0					